POINTS TO REMEMBER IN MATHEMATICS

By

Balraj Khurana

CLASS XI

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CH1. SET THEORY

I. A set is a well-defined collection of distinct objects. Well-defined collection means that there exists a rule with the help of which it is possible to tell whether a given object belongs or does not belong to given collection. Generally sets are denoted by capital letters A, B, C, X, Y, Z etc.

I. REPRESENTATION OF A SET

1. ROSTER FORM OR TABULAR FORM: In this form, we list all the member of the set within braces (curly brackets) and separate these by commas. For example, the set of all even numbers less than 10 and greater than 0 in the roster form is written as: A = {2,4, 6,8}

2. SET BUILDER FORM OR RULE FORM: In this form, we write a variable (say x) representing any member of the set followed by a property satisfied by each member of the set. A = $\{\boldsymbol{x}: \boldsymbol{x} = \boldsymbol{2n}\ \forall \boldsymbol{n} < \boldsymbol{5}, \boldsymbol{n \epsilon N}\}$the symbol ':' stands for the words" such that"and $\forall$ ***stands for the phrase "for all"*** .

II. TYPES OF SETS

- The *universal set* U contains every element in D.
- If A is a set in the domain D, A must be a *subset* of the universal set U, denoted as $A \subseteq U$. If A consists of some but not all elements, A is then called a *proper subset* of U, denoted as $A \subset U$.
- The number of elements of a set A is denoted as n (A)
- NULL/ VOID/ EMPTY SET –A set which has no element. It is denoted by Φ(phi). n (Φ) = 0 as it contains no element.
- SINGLETON SET **:** A set containing only one element is called Singleton Set.
- FINITE AND INFINITE SET: A set, which has finite numbers of elements, is called a finite set. Otherwise it is called an in finite set.
- SUBSET OF A SET : A set A is said to be a subset of the set B if each element of the set A is also the element of the set B. The symbol used is '⊆' i.e. A ⊆ B (xϵ A => x ϵB).
  - Each set is a subset of its own set.
  - Also a void set is a subset of any set.
  - If there is at least one element in B which does not belong to the set A, then A is a proper subset of set B and is denoted as A ⊂ B.
  - Total number of subsets of a finite set containing n elements is n(P(A)) = 2^n.
- DISJOINT SETS: If two sets A and B have no common elements i.e. if no element of A is in B and no element of B is in A, then A and B are said to be Disjoint Sets. Hence for Disjoint Sets A and B => n (A ∩ B) = 0.

III. OPERATIONS ON SETS

- UNION OF SETS: Union of two or more sets is the set of all elements that belong to any of these sets. The symbol used for union of sets is '∪' i.e. A∪B = Union of set A and set B = {x: x ϵA or x ϵB} (or both)} $A \cup B \supseteq A, A \cup B \supseteq B \Longrightarrow \quad n(A \cup B) \geq n(A), n(A \cup B) \geq n(B)$

- INTERSECTION OF SETS: It is the set of all the elements, which are common to all the sets. The symbol used for intersection of sets is '∩' i.e. A ∩ B = {x: x ϵA and xϵ B} $A \cap B \subseteq A, A \cup B \subseteq B \Longrightarrow \quad n(A \cap B) \leq n(A), n(A \cup B) \leq n(B)$

- **Complement** ("*not*") The *complement* of A consists of elements which do *not* belong to A, denoted by $\bar{A}$. n($\bar{A}$) $= n(U) - n(A)$
- DIFFERENCE OF SETS: The difference of set A to B denoted as A – B is the set of those elements that are in the set A but not in the set B i.e. A – B = {x: xϵ A and x ∉ B}
  Similarly B – A = {x: x B and x ∉ A}
  In general A–B ≠ B–A
- **Symmetric Difference of Two Sets:** For two sets A and B, symmetric difference of A and B is given by (A – B) ∪ (B – A) and is denoted by A Δ B.
- **Equality of Two Sets:** Sets A and B are said to be equal if A ⊆ B and B ⊆ A; we write A = B.

IV. Algebra of Sets

1. Universal law : A ∪ U = U A ∩ Φ = Φ
2. Law of complements : $A \cup A^c = U$, $A \cap A^c = \varphi$. $\Phi^C = U$; $U^C = \Phi$
3. Idempotent Law: For any set A, A ∪ A = A A ∩ A = A
4. Identity Law: For any set A, A ∪ ∅ = A A ∩ U = A
5. Commutative Law: For any two sets A and B A ∪ B = B ∪ A A ∩ B = B ∩ A
6. Associative Law: For any three sets A, B and C
   (A ∪ B) ∪ C = A ∪ (B ∪ C) A ∩ (B ∩ C) = (A ∩ B) ∩ C
7. Distributive Law: For any three sets A, B and C
   A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)
8. De Morgan's Law: For any two sets A and B: (A ∪ B)' = A' ∩ B' (A ∩ B)' = A' ∪ B'
9. Involute law : $(A^c)^c = A$

V. Addition theorem Let A, B and C be finite sets and U be the finite universal set, then

1. n (A U B) = n (A) + n (B) – n (A ∩ B)
   a. [n (A U B) is minimum or maximum according as n (A ∩ B) is maximum or minimum respectively.]
2. If A and B are disjoint, then n (A U B) = n (A) + n (B)
3. n (A –B) = n (A) – n (A ∩ B) i.e. n (A) = n (A – B) + n (A ∩ B)
4. n (A U B U C) = n (A) + n (B) + n (C) – n (A ∩ B) – n (B ∩ C) – n (A ∩ C) + n (A ∩ B ∩ C)

5. n (set of elements which are in exactly two of the sets A, B, C) = n (A ∩ B) + n (B ∩ C) + n (C ∩ A) –3n(A ∩ B ∩ C)
6. n(set of elements which are in at least two of the sets A, B, C) = n (A ∩ B) + n (A ∩ C) + n (B ∩ C) –2n(A ∩ B ∩ C)
7. n (set of elements which are in exactly one of the sets A, B, C)
   = n (A) + n (B) + n (C) – 2n (A ∩ B) – 2n (B ∩ C) – 2n (A ∩ C) + 3n (A ∩ B ∩ C)

   To solve the word problem represent the given facts by venn diagram and evaluate the required values by using the addition theorem.

CH 2. RELATIONS AND FUNCTIONS

I. CARTESIAN PRODUCT OF TWO SETS: If A and B are two finite sets then $\boldsymbol{A \times B = \{(a, b): a \epsilon A, b \in B\}}$.

- n($\boldsymbol{A \times B}$) = n(A).n(B)
- n[P($\boldsymbol{A \times B}$)] = $2^{n(A).n(B)}$

II. <u>RELATION</u>

- Let **A** and **B** be two sets. A **relation** between **A** and **B** is a collection of ordered pairs *(a, b)* such that $a \in$ **A** and $b \in B$
- If $R: A \to B$ is a relation from A to B, then $R \subseteq A \times B$
- If n(A) = m, n(B) = n ,then total number of relations from A to B is 2^{mn}.
- Domain of R = $\{a: (a, b) \in R\}$
- Range of R = $\{b: (a, b) \in R\}$
- Co-domain of R = B

III. <u>FUNCTIONS :</u>

- **<u>Definition</u> -** Any relation on A x B in which
  i. No two second elements have a common first element and
  ii. Every first element has a corresponding second element is called a function. It is also called **mapping**. A function is said to map an element x in its domain to an element y in its range. $f: A \to B$ *or* $f: x \to f(x)$ *then* $f(x) = y$ where y is a function of x.
- **DOMAIN** - The set of all the first elements of the ordered pairs of a function is called the domain
- **RANGE -** The set of all the second elements of the ordered pairs of a function is called the range
- **CODOMAIN** - If (a, b) is an ordered pair of the function $f: A \to B$ then the set B is called the Co-Domain. The range is a subset of the co-domain.

IV. **Some important facts** about a function from A to B:

- Every element in A is in the domain of the function; that is, every element of A is mapped to some element in the range. (If some element in S has no mapping (arrow), then the relation is *not* a function.)
- No element in the domain maps to more than one element in the range.
- The mapping is not necessarily onto; some elements of T may not be in the range.
- The mapping is not necessarily one-one; some elements of T may have more than one element of S mapped to them.
- S and T need not be disjoint.

V. Some special functions with their domain, range and nature

1. Polynomial function $p(x) = a_0 + a_1x+a_2x^2+\ldots+a_nx^n$; domain = R; range = R ; continuous
2. Constant Function f(x) = k domain = r ; range = {k} ; continuous
3. Identity function I(x) = x ; domain = R; range = R ; continuous
4. Exponential function $f(x) = e^x$ or a^x domain = R; domain = $(0, \propto)$; continuous
5. Logarithmic function f(x) = logx or In x domain = $(0, \propto)$: range = R ; continuous
6. Square root function f(x) = $\sqrt{x}$; domain = $(0, \propto)$; range = $(0, \propto)$; continuous.
7. Sine function - sin: R→ $[-1,1]; continuous$
8. Cosine function - cos: R→ $[-1,1]; continuous$
9. Tangent function - tan: R$-\left\{x: x = \frac{(2n+1)\pi}{2}\right\} \rightarrow R;$ continuous in its domain
10. Secant function - sec: R$-\left\{x: x = \frac{(2n+1)\pi}{2}\right\} \rightarrow R - (-1,1);$ continuous in its domain
11. Cosecant function - cosec: R$-\{x: x = n\pi, n \in Z\} \rightarrow R - (-1,1);$ continuous in its domain
12. Cotangent function - cot: R$-\{x: x = n\pi, n \in Z\} \rightarrow R;$ continuous in its domain
13. $Floor\ function\ \ x$ = Greatest integer that is less than or equal to x. domain= R, range = Z; discontinuous.
14. Ceiling function x = Least integer that is greater than or equal to x.domain= R; range = Z; discontinuous
15. Reciprocal function f(x) = $\frac{1}{x}$; domain = R - {o};range = R - {o} continuous in R+ and R-
16. Modulus function f(x) = $|x| = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases}$; Domain = R; Range = R + ; continuous.
17. Signum function f(x) = $\begin{cases} \frac{|x|}{x}, & \forall x \neq 0 \\ 0, & x = 0 \end{cases} = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$; domain = R ;range = {-1 , 0 ,1}; discontinuous.

VI. **COMPOSITION OF FUNCTIONS -** function composition is the application of one function to the results of another. For instance, the functions $f: X \to Y$ and $g: Y \to Z$ can be *composed* by computing the output of g when it has an input of $f(x)$ instead of x. A function $g \circ f: X \to Z$ defined by $(g \circ f)(x) = g(f(x))$ for all x in X.

- The composition of functions is always associative. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$,
- The functions g and f are said to commute with each other if $g \circ f = f \circ g$.

CH 3. TRIGONOMETRIC FUNCTIONS

I. MEASUREMENT OF ANGLES There are two systems used for the measurement of angles.

- Sexagesimal system:Here a right angle is divided into 90 equal parts known as degrees. Each degree is divided into 60 equal parts called minutes and each minute is further divided into 60 equal parts called seconds.
- 90 degrees (or 90°) = 1 right angle
- 60 minutes (or 60') = 1 degree (or 1°)
- 60 seconds (or 60") = 1 minute (or 1')
- **Circular Measurement:** In this system a unit called **'Radian'** is defined as one radian corresponds to the angle subtended by arc of length 'r' at the centre of the circle. One Radian (1^c) = arc length of magnitude 'r' / Radius of circle (r)
- **π radians = 180°**

II. ALGEBRAIC IDENTITIES

1. $(a + b) \cdot (a - b) = a^2 - b^2$
2. $a^2 + b^2 = (a + b)^2 - 2ab = (a - b)^2 + 2ab$
3. $(a + b)^2 = a^2 + 2ab + b^2 = (a - b)^2 + 4ab$
4. $(a - b)^2 = a^2 - 2ab + b^2 = (a + b)^2 - 4ab$
5. $a^3 + b^3 = (a + b) \times (a^2 - ab + b^2) = (a + b)^3 - 3ab \times (a + b)$
6. $a^3 - b^3 = (a - b) \times (a^2 + ab + b^2) = (a - b)^3 + 3ab \times (a - b)$
7. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab \times (a + b)$
8. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab \times (a + b)$
9. $(x + a) \times (x + b) = x^2 + x \times (a + b) + ab$
10. $(x - a) \times (x + b) = x^2 + x \times (b - a) - ab$
11. $(x - a) \times (x - b) = x^2 - x \times (a + b) + ab$
12. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$
13. $(a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a)$
14. $a^4 - b^4 = (a - b) \times (a + b) \times (a^2 + b^2)$
15. $a^6 - b^6 = (a + b) \times (a^2 - ab + b^2) \times (a - b) \times (a^2 + ab + b^2)$

17. $a^6 + b^6 = (a^2 + b^2) \times (a^4 - a^2b^2 + b^4)$
18. $a^4 + a^2b^2 + b^4 = (a^2 - ab + b^2) \times (a^2 + ab + b^2)$
19. $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 3bc - 2ac$
20. $a^3+b^3+c^3-3abc = (a+b+c)\times(a^2+b^2+c^2-ab-bc-ac)$

III. INDICES FORMULAS:The formulas involving relations between variables and their powers or powers and indices are:

1. $x^m \times x^n = x^{m+n}$
2. $x^m \times x^n \times \ldots \times x^p = x^{m+n+\ldots+p}$
3. $x^m \div x^n = x^{m-n}$
4. $x^m \div x^n \div \ldots \div x^p = x^{m-n-\ldots-p}$
5. $(x^m)^n = x^{m\times n}$
6. $((x^m)^n)^o) = x^{m\times n\times o}$
7. $x^0 = 1$
8. $x^{-m} = \frac{1}{x^m}$
9. $x^m = \frac{1}{x^{-m}}$
10. $x^{\frac{m}{n}} = \sqrt[n]{x^m}$
11. $\left(\frac{x^a}{y^b}\right)^c = \frac{x^{ac}}{y^{bc}}$
12. $\frac{x^m}{y^m} = \left(\frac{x}{y}\right)^m$
13. $\sqrt[m]{\frac{x^a}{y^b}} = \frac{x^{\frac{a}{m}}}{y^{\frac{b}{m}}}$
14 $x^{\frac{p}{q}} = \sqrt[q]{x^p} = (\sqrt[q]{x})^p$
15 $\sqrt[m]{\frac{x}{y}} = \frac{\sqrt[m]{x}}{\sqrt[m]{y}}$
16. If, $a^x = a^y$, then x = y.
17. If, $a^x = b^x$, then a = b

18. $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$ provided that a , b and a*b are not negative numbers.

IV. <u>TRIGONOMETRIC IDENTITIES</u>

1. Reciprocal identities

$$\sin u = \frac{1}{\csc u} \quad \cos u = \frac{1}{\sec u} \quad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \quad \sec u = \frac{1}{\cos u} \quad \cot u = \frac{1}{\tan u}$$

2. Pythagorean Identities:

$$\sin^2 u + \cos^2 u = 1 \quad 1 + \tan^2 u = \sec^2 u \quad 1 + \cot^2 u = \csc^2 u$$

3. Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

4. Co-Function Identities

$\sin(\frac{\pi}{2}-u)=\cos u \quad \cos(\frac{\pi}{2}-u)=\sin u \quad \tan(\frac{\pi}{2}-u)=\cot u$

$\csc(\frac{\pi}{2}-u)=\sec u \quad \sec(\frac{\pi}{2}-u)=\csc u \quad \cot(\frac{\pi}{2}-u)=\tan u$

5. Even-Odd Identities:

$\sin(-u)=-\sin u \quad \cos(-u)=\cos u \quad \tan(-u)=-\tan u$

$\csc(-u)=-\csc u \quad \sec(-u)=\sec u \quad \cot(-u)=-\cot u$

6. Sum-Difference Formulas

Sin (u$\pm v$) = $\sin u\cdot\cos v\pm\cos u\cdot\sin v$

cos (u$\pm v$) = $\cos u\cdot\cos v\mp\sin u\cdot\sin v$

tan(u$\pm v$) = $\frac{\tan u\pm\tan v}{1\mp\tan u\cdot\tan v}$

- $sin2u = 2sinucosu = \frac{2tanu}{1+tan^2u}$
- cos2u = $\cos^2$u – $\sin^2$u = 2 $\cos^2$u – 1 = 1 – 2$\sin^2$u = $\frac{1-tan^2u}{1+tan^2u}$
- Sin3u= 3sinu – 4sin3u
- Cos3u = 4cos3u – 3cosu

7. Multiple Angle Formulas

- $\tan(2u) = \frac{2\tan u}{1-\tan^2 u}$
- Tan3u = $\frac{3tanu-tan^3u}{1-3tan^2u}$

8. Power-Reducing/Half Angle Formulas

- $\sin^2 u = \frac{1-\cos 2u}{2}$
- $\cos^2 u = \frac{1+\cos 2u}{2}$
- $\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$
- Sin3u = $\frac{3sinu-sin3u}{4}$
- cos3u = $\frac{3cosu+cos3u}{4}$

9. Product-to-Sum Formulas

- sinu.sinv = $\frac{1}{2}[cos(u-v)-cos(u+v)]$
- Sinu.cosv = $\frac{1}{2}[sin(u+v)+sin(u-v)]$
- cosu.sinv = $\frac{1}{2}[sin(u+v)-cos(u-v)]$

10. Sum-to-Product Formulas

- sinu + sinv = $2sin\frac{(u+v)}{2}cos\frac{(u-v)}{2}$
- sinu - sinv = $2cos\frac{(u+v)}{2}sin\frac{(u-v)}{2}$
- cosu + cosv = $2cos\frac{(u+v)}{2}cos\frac{(u-v)}{2}$
- cosu - cosv = $2sin\frac{(u+v)}{2}cos\frac{(v-u)}{2}$

11. **PROPERTIES OF TRIANGLES** In a triangle ABC, the angles are denoted by capital letters A, B, and C and the lengths of the sides opposite to these angles are denoted by small letters a, b, and c respectively.

- **Sine rule:** sinA/a = sinB/b = sinC/c = 1/2R, where R is the radius of the circumcircle of the ΔABC.
- **Cosine rule:** cosA = $\frac{b^2+c^2-a^2}{2bc}$, cosB = $\frac{c^2+a^2-b^2}{2ac}$, cosC = $\frac{a^2+b^2-c^2}{2ab}$

- **Projection rule:** a = b cosC + c cosB, b = c cosA + a cosC, c = a cosB + b cosA.
- **Napier's analogy:** tan $\frac{B-C}{2}$= $\frac{b-c}{b+c}$cot $\frac{A}{2}$, tan $\frac{C-A}{2}$= $\frac{c-a}{c+a}$cot $\frac{B}{2}$, tan $\frac{A-B}{2}$= $\frac{a-b}{a=b}$cot $\frac{C}{2}$

V. **Periodic Functions** if a function repeats its value after a fixed interval it is called periodic function

The periodicity of sine and cosine function is 2π and that of tangent function is π such that $\sin(\theta + 2\pi) = \sin\theta$; $\cos(\theta + 2\pi) = \cos\theta$; $\tan(\theta + \pi) = \tan\theta$;In general, we have for all angles θ:

$$\cos(\theta + 2n\pi) = \cos\theta,\ \sin(\theta + 2n\pi) = \sin\theta,\ n = 0, \pm 1, \pm 2, \ldots \qquad (2)$$

VI. Trigonometric equations

- **A Solution** is a value of the angle which satisfies a given trigonometric equation.
- Solution in a particular interval, such as $0 \le x \le 2\pi$ are usually known as **"primary solution"**.
- A **general solution** is that formula which lists all possible solutions.
- steps to solve : Some standard procedures are given below.
    1) **Factoring (2) Squaring both sides (3) Expressing various functions in terms of single function .**

I. Convert the equation in terms of one function of one angle.
II. Write the equation as one trig function of an angle θ equals a constant.
III. Write down the possible value(s) for the angle.(α) when θ is the variable and $\alpha$is the smallest possible angle between 0 to 2π, called the reference angle.
IV. Compare with the following equations to write the general solution accordingly.

Equation	Solution
Sin θ = 0	$\theta = n\pi$
cosθ = 0	$\theta = (2n+1)\frac{\pi}{2}$
tan θ = 0	$\theta = n\pi$
$\sin\theta = \sin\alpha$	$\theta = n\pi + (-1)^n\alpha$
$\cos\theta = \cos\alpha$	$\theta = 2n\pi \pm \alpha$
$\tan\theta = \tan\alpha$	$\theta = n\pi + \alpha$

CH5. Complex Numbers

I. Definitions

1. Iota is defined by $i^2 = -1$.
2. $i^1 = i$, $i^2 = -1$. $i^3 = -i$, $i^4 = 1$.
3. **In general, $i^{4n} = 1$, $i^{4n+1} = i$, $i^{4n+2} = -1$, $i^{4n+3} = -i$ for an integer n.**
4. Every complex number has the ``Standard Form" a + b i , for some real *a* and *b*.
5. If $z = a + bi$ is a complex number and *a* and *b* are real, we say that *a* is the real part of *z* and that *b* is the imaginary part of *z* and we write Re z = a and Imz = b
6. A complex number is said to be purely real if Im(z) = 0, and is said to be purely imaginary if Re (z) = 0. The complex number 0 = 0 + i0 is both purely real and purely imaginary.
7. Since a real number a can be written as a + i.0, therefore every real number can be considered as a complex number whose imaginary part is zero. Thus the set R of real numbers is a proper subset of the complex numbers C.

II. Operations on complex numbers

1. Two complex numbers are said to be equal if and only if their real parts and imaginary parts are separately equal i.e. a + ib = c + id implies a = c and b = d. However, there is no order relation between complex numbers and the expressions of the type a + ib < (or >) c + id are meaningless.
2. For real *a* and *b*, a + bi = 0 if and only if a = 0 = b
3. (a + bi)$\pm$(c + di)= $(a \pm c) + (b \pm d)i$
4. (a + bi)(c + di)= (ac – bd) + (bc+ad) i
5. $\frac{a+bi}{c+di} = \frac{ac+bd}{c^2+d^2} + i\,\frac{bc-ad}{c^2+d^2}$
6. $Additive\ inverse\ of\ z: \ -z = -a - bi$
7. $Multiplicative\ inverse\ of\ z: \quad z^{-1} = \frac{1}{z} = \frac{\bar{z}}{|z|^2}$
8. The conjugate of the complex number z = a + ib is defined to be a – ib and is denoted by $\bar{z}$. In other words is the mirror image of z in the real axis.

III. Properties of Conjugate

- $|z| = |\bar{z}|$
- z + $\bar{z}$ = 2Re(z). z – $\bar{z}$ = 2i Im(z). Also Re (z) = $\frac{(z+\bar{z})}{2}$, lm(z) = $\frac{(z-\bar{z})}{2i}$.
- If z is purely real z = $\bar{z}$. whenever we have to show a complex number purely real we use this property.
- If z is purely imaginary z+ $\bar{z}$= 0, whenever we have to show that a complex number is purely imaginary we use this property
- $z\bar{z} = |z|^2 = |\bar{z}|^2$
- $\overline{z_1 \pm z_2} = \bar{z_1} \pm \bar{z_2}$. In general $\overline{z_1 \pm z_2 \pm \cdots \pm z_n} = \bar{z_1} \pm \bar{z_2} \pm \cdots \pm \overline{z_n}$
- $\overline{z_1.z_2} = \bar{z_1}.\bar{z_2}$. In general $\overline{z_1.z_2.\ldots.z_n} = \bar{z_1}.\bar{z_2}.\ldots.\overline{z_n}$

- $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

IV. The magnitude or *modulus* of a complex number $z = x + y\,i$ is denoted $|z|$ and defined as

$$|z| = r = \sqrt{x^2 + y^2} = \sqrt{(Rez)^2 + (Imz)^2}$$

V. Properties of modulus

i. $|z| = |\bar{z}|$
ii. $z\bar{z} = |z|^2$
iii. $|z_1 z_2| = |z_1||z_2|$
iv. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$
v. $|z_1 + z_2| \leqslant |z_1| + |z_2|$
vi. $|z_1 + z_2| \geqslant |z_1| - |z_2|$
vii. $|z_1 - z_2| \geqslant |z_1| - |z_2|$

VI. POLAR REPRESENTATION OF Z = x + iy : Z = r(cosθ + i sin θ),where r is the modulus and θ is the argument of z . If $\alpha = \tan^{-1}\left|\frac{y}{x}\right|$ is the reference angle, then argument of z is given by

a. $\theta = \alpha$ when $x > 0, y > 0$ (z lies in I quadrant)
b. $\theta = \pi - \alpha$ when $x < 0, y > 0$ (z lies in II quadrant)
c. $\theta = \pi + \alpha$ or $-(\pi - \alpha)$ when $x < 0, y < 0$ (z lies in III quadrant)
d. $\theta = 2\pi - \alpha$ or $-\alpha$ when $x > 0, y < 0$ (z lies in IV quadrant)

VII. **TO FIND THE SQUARE ROOT OF THE COMPLEX NUMBER a + ib WHERE a AND b (≠0) ARE REAL**

- Let $\sqrt{a + ib} = x + iy$ where x and y are real.
- Then, $a + ib = (x + iy)^2 = x^2 + i^2y^2 + 2x.\,iy = x^2 - y^2 + i.\,2xy$
- Equating real and imaginary parts we get, $x^2 - y^2 = a$ (1) and $2xy = b$ (2)
- Now $(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2 = a^2 + b^2$
- Hence $x^2 + y^2 = +\sqrt{a^2 + b^2}$ (Since x, y both are real) (3)
- Now adding equations (1) and (3) we get, $2x^2 = a + \sqrt{a^2 + b^2}$ or $x = \pm\left(\frac{a + \sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}$
- Now equation (3) - equation (1) gives, $2y^2 = \sqrt{a^2 + b^2} - a$ or $y = \pm\left(\frac{\sqrt{a^2+b^2} - a}{2}\right)^{\frac{1}{2}}$

From equation (2) it is clear that both x and y will have the same signs (either both positive or both negative) when b is positive; and x and y will have different signs (one positive and the other negative) when b is negative.

- Hence if b > 0 then the square roots of (a + ib) are: $\pm\left[\left(\frac{a + \sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{\sqrt{a^2+b^2} - a}{2}\right)^{\frac{1}{2}}\right]$

- And if b < 0 then the square roots of (a + ib) are: $\pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{\sqrt{a^2+b^2}-a}{2}\right)^{\frac{1}{2}}\right]$

CH 6. LINEAR INEQUALITIES

I. The procedure for solving linear inequalities in one variable is similar to solving basic equations. We need to be careful about the **sense** of the equality when multiplying or dividing by negative numbers.

II. **Properties of Inequalities** Let a, b and c be real numbers.

1. Transitive Property
   If a < b and b < c then a < c
2. Addition Property
   If a < b then a + c < b + c
3. Subtraction Property
   If a < b then a - c < b - c
4. Multiplication Property
   - If a < b and c is positive then c*a < c*b
   - If a < b and c is negative c*a > c*b

- If each inequality sign is reversed in the above properties, we obtain similar properties.
- If the inequality sign < is replaced by≤ (less than or equal) or the sign > is replaced by ≥ (greater than or equal), we also obtain similar properties.

III. Solving an inequality in the variable *x* means finding all the values of *x* for which the inequality is true.

- To solve a linear inequality in one variable, use the properties of inequalities to isolate the variable.
- When each side of an inequality is multiplied or divided by a negative number, the direction of the inequality symbol must be reversed.
- The solution set to be represented in the number line and in the form of sets.

IV. Two inequalities that have the same solution set are said to be equivalent .

V. **Inequalities Involving Absolute Value** Let *x* be a variable or an algebraic expression and let *a* be a real number such that $a \neq 0$.

- The solutions of $|x| < |a|$ are all values of *x* that lie between - *a* and *a* .
- The solutions of $|x| > |a|$ are all values of *x* that are less than - *a* or greater than a

VI. Linear inequalities of the type $\frac{ax+b}{cx+d} \leq c$ or $\frac{ax+b}{cx+d} \geq c$

1. Bring constant term to the same side of the variable.
2. Take lcm.
3. Make coefficient of x positive in both numerator and denominator.
4. Equate numerator and denominator separately to zero to get the critical points
5. Mark these critical points on the number line to divide it into sub-intervals.
6. Mark the sub-intervals on the number line alternatively positive or negative, starting from the right most sub-interval.

7. Refer the sign of inequality obtained after step 3 the corresponding subintervals will give the solution set.

VII. The solution of a linear inequality in two variables like $Ax + By > C$ is an ordered pair (x, y) that produces a true statement when the values of x and y are substituted into the inequality.
VIII. The graph of an inequality in two variables is the set of points that represents all solutions to the inequality.
IX. A linear inequality divides the coordinate plane into two halves by a boundary line where one half represents the solutions of the inequality. The boundary line is dashed for > and < and solid for ≤ and ≥. The half-plane that is a solution to the inequality is usually shaded
X. The STEPS to solve a system of Linear inequalities in two variables:

- Write the given inequalities in different columns.
- Write the corresponding equations.
- Find the solution set for each one.
- Plot the graph of the equation.
- Identify whether the line representing the equation is to be included or not. For strict inequalities draw dotted line and for simple inequalities draw bold line
- Identify the HALF PLANE representing the inequality by substituting the value of any suitable point from one half plane in the inequality. If it satisfies the inequality shade the region containing that point otherwise shade the other half plane (not containing the point)
- Repeat these steps for each inequality in the system.
- The overlapping shaded region is the feasible region representing the solution set for the system of inequalities

CH7. **PERMUTATIONS** & **COMBINATIONS**

I. PERMUTATIONS (ARRANGEMENT OF OBJECTS)

The number of permutations of n objects, taken r at a time, is the total number of arrangements of r objects, selected from n objects where the order of the arrangement is important.

- **Without Repetition:**

(a) Arranging n objects, taken r at a time is equivalent to filling r places from n things.
The number of ways of arranging = The number of ways of filling r places
$= n(n-1)(n-2) \ldots (n-r+1)$
$= n(n-1)(n-2)\ldots(n-r)+1\ ((n-r)!)/(n-r)! = n!/(n-r)! = {}^nP_r$
(b) The number of arrangements of n different objects taken all at a time $= {}^np_n = n!$

- **With Repetition:**

(a) The number of permutations (arrangements) of n different objects, taken r at a time, when each object may occur once, twice, thrice…. upto r times in any arrangement
= The number of ways of filling r places where each place can be filled by any one of n objects .
= The number of ways of filling r places by n ways each $= (n)^r$
(b) The number of arrangements that can be formed using n objects out of which p are identical (and of one kind), q are identical (and of another kind), r are identical (and of another kind) and the rest are distinct is n! / p!q!r!.

II. CIRCULAR PERMUTATIONS **-** There are arrangements in closed loops also, called as circular arrangements. Suppose n persons (a_1, a_2, a_3,…,a_n) are to be arranged around a circular table. The total number of circular arrangements of n persons is n!/n = (n – 1)!.

- A combination of ***n*** taken ***r*** at a time is defined as a selection of ***r*** out of the ***n*** items without regard to the order. The total number of all the possible combinations is denoted as:

$$C(n,r) = {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

- **RESTRICTED SELECTION / ARRANGEMENT**
  1. The number of ways in which r objects can be selected from n different objects if k particular objects are
     a) always included = ${}^{n-k}C_{r-k}$
     b) never included = ${}^{n-k}C_r$
  2. The number of arrangements of n distinct objects taken r at a time so that k particular objects are
     a) always included = ${}^{n-k}C_{r-k}$ .r!
     b) never included = ${}^{n-k}C_r$ .r!
  3. **Some Results Related to ${}^nC_r$:**
     i. ${}^nC_r = {}^nC_{n-r}$
     (i) If ${}^nC_r = {}^nC_k$, then r = k or n-r =k
     (ii) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
     (iii) ${}^nC_r = {}^{n-1}C_{r-1}$
     (iv) ${}^nC_r/{}^nC_{r-1} = n-r+1/r$
     (v) If n is even , ${}^nC_r$ is greatest for r = n/2
     (vi) If n is odd, ${}^nC_r$ is greatest for r = n–1/2, 2+1/2

III. ADDITION PRINCIPLE

If one experiment has n possible outcomes and another has m possible outcomes, then there are (m + n) possible outcomes when exactly one of these experiments is performed.
In other words, if a job can be done by n different methods and for the first method there are a_1 ways, for the second method there are a_2 ways and so on . . . for the nth method, a_n ways, then the number of ways to get the job done is ($a_1 + a_2 + ... + a_n$).

IV. MULTIPLICATION PRINCIPLE If one experiment has n possible outcomes and another experiment has m possible outcomes, then there are m × n possible outcomes when both of these experiments are performed.

CH 8. BINOMIAL THEOREM

- Binomial Theorem : For a positive integer n , the expansion is given by

$(a+x)^n = {}^nC_0a^n + {}^nC_1a^{n-1} x + {}^nC_2 a^{n-2} x^2 + . . . + {}^nC_r a^{n-r} x^r + . . . + {}^nC_nx^n = \sum^n_{r=0} {}^nC_r a^{n-r} x^r$ where ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, . . . , ${}^nC_n$ are called Binomial co-efficients.

- Similarly $(a - x)^n = {}^nC_0a^n - {}^nC_1a^{n-1} x + {}^nC_2 a^{n-2} x^2 - . . . + (-1)^r {}^nC_r a^{n-r} x^r + . . . +(-1)^n {}^nC_nx^n = \sum^n_{r=0} (-1)^r {}^nC_r a^{n-r} x^r$

Replacing a = 1, we get

- $(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_r x^r + \ldots + {}^nC_nx^n$
- $(1 - x)^n = {}^nC_0 - {}^nC_1x + {}^nC_2x^2 - \ldots + (-1)^r\, {}^nC_r x^r + \ldots + (-1)^n\, {}^nC_nx^n$

Properties of Binomial Theorem:

* There are (n+1) terms in the expansion of $(a + x)^n$.
* Sum of powers of x and a in each term in the expansion of $(a + x)^n$ is constant and equal to n.
* The general term in the expansion of $(a+x)^n$ is $(r+1)^{th}$ term given as $T_{r+1} = {}^nC_r\, a^{n-r}\, x^r$
* The p^{th} term from the end of $(a+x)^n = (n - p + 2)^{th}$ term from the beginning or the p^{th} term of the expansion $(x+a)^n$
* Coefficient of x^r in expansion of $(a + x)^n$ is ${}^nC_r\, a^{n-r}\, x^r$.
* ${}^nC_x = {}^nC_y$ iff x = y or x + y = n.
* In the expansion of $(a + x)^n$ and $(a - x)^n$, x^r occurs in $(r + 1)^{th}$ term.

- **GREATEST BINOMIAL COEFFICIENT**

In the binomial expansion of $(1 + x)^n$, when n is even, the greatest binomial coefficient is given by ${}^nC_{n/2}$.Similarly if n be odd, the greatest binomial coefficient will be ${}^nC_{\frac{n+1}{2}}$ and ${}^nC_{\frac{n-1}{2}}$ both being equal.

- MIDDLE TERM of $(a+x)^n$

a) If n is odd, total no of terms n+1 is even, so there are two middle terms $\left(\frac{n+1}{2}\right)^{th}$ *and* $\left(\frac{n+1}{2}+1\right)^{th}$ terms.

b) If n is even, total no of terms n+1 is odd, so there will be only one middle terms $\left(\frac{n}{2}+1\right)^{th}$ term.

Note: In order to find a particular term always start from the expression for the general term $T_{r+1} = {}^nC_r\, a^{n-r}\, x^r$ simplify to get the value of r.

CH 9. SEQUENCE AND SERIES

- A Sequence is a set of things or items or members that are in order.If the sequence goes on forever it is called an infinite sequence, otherwise it is a finite sequence

- Series When you sum up an infinite sequence it is called a "Series"

I. Arithmetic Progression- An **arithmetic progression** is a sequence of numbers such that the difference of any two successive terms of the sequence is a constant.

- If the initial term of an arithmetic progression is a_1 and the common difference of successive members is *d*, then the *n-th* term of the sequence is given by $a_n = a_1 + (n - 1)d,\ n = 1, 2, \ldots$
- The sum S of the first n terms
  $S_n = \frac{1}{2}(a_1 + a_n)n$, where a_1 is the first term and a_n the last. or $S_n = \frac{n}{2}(2a_1 + d(n-1))$
- If a, b, c are in A.P. then b is called the arithmetic mean(A.M.) between a and c, $b = \frac{a+c}{2}$.

II. Geometric Progression- A **geometric progression** (G.P.) is a sequence of numbers such that the ratio of any two successive terms of the sequence is a constant.

- If the initial term of a geometric progression is *a* and the common ratio of successive terms is *r*, then the n^{th} term of the sequence is given by $a_n = a\, r^{(n-1)}, n = 1, 2, ...$
- The sum S of the first n terms

$S_n = a + ar + ar^2 + \cdots + ar^{n-1} = a\frac{(r^n-1)}{(r-1)}, |r| > 1 \text{ or } \frac{a(1-r^n)}{1-r}, |r| < 1.$

- Sum to infinity of a G.P. a, ar, ar^2, ar^3,... is S = $\frac{a}{1-r}$
- If a, b, c are in G.P. then b is called the geometric mean(G.M.) between a and c, b = $\sqrt{ac}$.

III. A.M.> G.M.

IV. Special series

i. $1 + 2 + 3 + \cdots + (n-1) + n = \frac{n(n+1)}{2}$.

ii. $1^2 + 2^2 + 3^2 + \cdots + (n-1)^2 + n^2 = \frac{n(n+1)(2n+1)}{6}$.

iii. $1^3 + 2^3 + 3^3 + \cdots + (n-1)^3 + n^3 = \frac{n^2(n+1)^2}{4}$.

Note: In order to insert n A.M.s or n G.M.s between two given numbers a and b, take b as the $(n+2)^{th}$ term of the corresponding A.P. or G.P. respectively .then evaluate d or r in term of a,b and n.

CH 10. STRAIGHT LINES

- Points are defined as ordered triples of real numbers and the distance between points $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$ is defined by the formula
  $P_1P_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
- Distance of the point P(x,y,z) from the origin is $\sqrt{x^2 + y^2 + z^2}$.
- Section formula - the coordinate of a point R dividing the line segment PQ joining P $(\vec{a})(x_1, y_1, z_1)$ and Q$(\vec{b})$ (x_2, y_2, z_2) in the ratio m : n is given by

Internal division

Vector form $\vec{r} = \frac{m\vec{b}+n\vec{a}}{m+n}$, Cartesian form $x = \frac{mx_2+nx_1}{m+n}, y = \frac{my_2+ny_1}{m+n}, z = \frac{mz_2+nz_1}{m+n}$

External division

Vector form $\vec{r} = \frac{m\vec{b}-n\vec{a}}{m-n}$, Cartesian form $x = \frac{mx_2-nx_1}{m-n}, y = \frac{my_2-ny_1}{m-n}, z = \frac{mz_2-nz_1}{m-n}$

- Midpoint formula: Vector form $\vec{r} = \frac{\vec{a}+\vec{b}}{2}$, Cartesian form $x = \frac{x_1+x_2}{2}, y = \frac{y_1+y_2}{2}, z = \frac{z_1+z_2}{2}$

I. Inclination of a line is the angle θ made by the line with the positive direction of x axis.

II. The Gradient or Slope of a straight line is defined as the tangent of its inclination. It shows how steep a straight line is. The method to calculate the Gradient is: Divide the change in height by the change in horizontal distance. Sometimes the horizontal change is called "run", and the vertical change is called "rise" or "fall":Gradiant = m$= tan\theta = \frac{\textit{change in height}}{\textit{change in horizontal distance}} = \frac{rise}{run} = \frac{y_2-y_1}{x_2-x_1}$

- Line is going up then gradient is positive,
- Line is going down then gradient is negative
- A Horizontal line has a gradient of zero.
- A Vertical line's gradient is "undefined".

III. Angle θ between two lines with slopes m_1 and m_2 is given by $\tan\theta = \left|\frac{m_2-m_1}{1+m_2m_1}\right|$, where +ve value gives the acute angle and –ve value gives the obtuse angle between the lines.

- If the lines are parallel , $m_1 = m_2$
- If the lines are perpendicular, $m_1 . m_2 = -1$

IV. Equation of a line

- Slope - intercept form : The equation of a line with slope m and y-intercept c is y = m x + c
  The equation of a line with slope m and x-intercept d is y = m (x – d)
- Slope point form : Passing through the point $Q(x_1,y_1)$and with slope m is $y – y_1 = m(x – x_1)$
- Two point form: the equation of a line passing through the two points $A(x_1,y_1)$ and $B(x_2,y_2)$ is $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$
- Equation of a line in intercept form: The equation of the line with x - intercept a and y – intercept b is $\frac{x}{a} + \frac{y}{b} = 1$
- Equation of a line in normal form: The equation of the line whose length of the perpendicular from the origin is p and the perpendicular makes an angle ω with the positive direction of x axis is $x\cos\omega + y\sin\omega = p$
- General equation of a line: A linear equation Ax + By + C = 0 always represents a straight line provided A,B are not simultaneously 0.
  CaseI: When A = 0, the equation reduces to By + C = 0 or y = -C/B which represents a line parallel to x axis.
  Case II:When B = 0, the equation reduces to Ax+ C = 0 or x = -C/A which represents a line parallel to y axis.
- To convert general equation Ax + By + C= 0 into other forms:
- Normal form: $\frac{A}{\pm\sqrt{A^2+B^2}}x + \frac{B}{\pm\sqrt{A^2+B^2}}y = -\frac{C}{\pm\sqrt{A^2+B^2}}$ Where $\cos\alpha = \frac{A}{\pm\sqrt{A^2+B^2}}$, $\sin\alpha = \frac{B}{\pm\sqrt{A^2+B^2}}$, $p = \frac{-C}{\pm\sqrt{A^2+B^2}}$
- Slope intercept form: $y = -\frac{A}{B}x - \frac{C}{B}$, where Slope m = - A/B and y-intercept c = -C/B.

- Intercept form : $\frac{x}{-C/A} + \frac{y}{-C/B} = 1$, where x-intercept = –C/A , y-intercept = –C/B

V. Distance of a point (x_1,y_1) from a line Ax+By+C=0 is given by $d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$

VI. Distance between two parallel lines :If $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ be two parallel lines then distance between them is given by

$d = \left|\frac{c_2 - c_1}{\sqrt{A^2+B^2}}\right|$ OR $d = \left|\frac{c_2 - c_1}{\sqrt{1+m^2}}\right|$ if the equations of the parallel lines are $y = mx + c_1$ & $y = mx + c_2$.

VII. The angle between two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$: If θ be the angle between the given lines then

$$\tan\theta = \left|\frac{a_1b_2 - a_2b_1}{a_1a_2 + b_1b_2}\right|$$

VIII. The equation of angular bisector of the angle between two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is given by $\left|\frac{a_1x + b_1y + c_1}{\sqrt{a_1{}^2+b_1{}^2}}\right| = \left|\frac{a_1x + b_1y + c_1}{\sqrt{a_1{}^2+b_1{}^2}}\right|$ or $\frac{a_1x + b_1y + c_1}{\sqrt{a_1{}^2+b_1{}^2}} = \pm\frac{a_1x + b_1y + c_1}{\sqrt{a_1{}^2+b_1{}^2}}$ Equation corresponding to the =ve sign will give bisector of the acute angle between the lines and the equation corresponding to the –ve sign will give the bisector of the obtuse angle between the lines.

CH 11. CONIC SECTION

A conic section is defined as a locus of a moving point the ratio of its distance from a fixed point to its distance from a fixed line is fixed. The fixed point is called the focus, the fixed line is called the directrix and the fixed ratio is called the eccentricity (e).

- e = 1 gives a parabola.
- e< 1 gives an ellipse.
- e > 1 gives a hyperbola.
- e = 0 gives a circle.

1. A general equation of 2nd degree always represents a conic section $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where A, B , C are not all zero.

2. a circle is defined as the locus of a moving point which moves in such a way that its distance from a fixed point is constant. The **fixed point** is called the **centre of the circle** and the **constant distance,** the **radius of the circle**.
   i. A circle is a special kind of ellipse with the following features

- a = b

- c = 0 (where ce is the distance from the center to a focus)
- the foci are at the same point so there is really only 1 focus i.e. the centre
- the minor axis is the same length as the major
- the eccentricity of the ellipse is zero

ii. Circle formula:

- Central form: $(x-h)^2+(y-k)^2=r^2$, centre at (h,k) , radius r.
- Standard form: $x^2 + y^2 = r^2$, centre at (0,0) , radius r.
- Diametrical form: $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$ where end points of a diameter are (x_1, y_1) and (x_2 , y_2)
- Parametric form: $x = rcos\theta,\ \ y = rsin\theta$ when centre at the origin and radius r units or $x = h + rcos\theta,\ \ y = k + rsin\theta$ when centre at (h,k) and radius r units.
- General form : $x^2 + y^2 + 2gx + 2fy + c = 0$, centre at (-g, - f), radius = $\sqrt{g^2 + f^2 - c}$

3. Parabola : it is the locus of a moving point whose distance from a fixed point (called foci) is equal to its distance from a fixed line (called the directrix).

- The parametric equations of the parabola $y^2 = 4ax$ are $x = at^2$, $y = 2at$, where t is the parameter.
- Parabola formula:

equation	nature	vertex	axis	focus	directrix
$y^2 = 4ax$	Opening rightward	(0,0)	x- axis	(a,0)	x =$-a$
$y^2 = -4ax$	Opening leftward	(0,0)	x- axis	(- a,0)	x =$+a$
$x^2 = 4ay$	Opening upward	(0,0)	y- axis	(0, a)	y=$-a$
$x^2 = -4ay$	Opening downward	(0,0)	y- axis	(0, -a)	y= a
$(y - k)^2 = 4a(x - h)$	Opening rightward	(h,k)	y = k	(h+ a, k)	x = h- a
$(y - k)^2 = -4a(x - h)$	Opening leftward	(h,k)	y = k	(h - a, k)	x = h + a
$(x - h)^2 = 4a(y - k)$	Opening upward	(h,k)	x = h	(h, k + a)	y = k- a
$(x - h)^2 = -4a(y - k)$	Opening downward	(h,k)	x = h	(h, k - a)	y = k + a

4. An ellipse is the set of all points in a plane such that the sum of the distances from T to two fixed points F_1 and F_2 is a given constant, K. $TF_1 + TF_2 = K$ where F_1 and F_2 are both foci(plural of focus) of the ellipse.

- The major axis is the segment that contains both foci and has its endpoints on the ellipse. These endpoints are called the vertices. The midpoint of major axis is the center of the ellipse.
- The minor axis is perpendicular to the major axis at the center, and the endpoints of the minor axis are called co-vertices.
- The vertices are at the intersection of the major axis and the ellipse.
- The co-vertices are at the intersection of the minor axis and the ellipse.

- Ellipse formula:

Nature of ellipse	Horizontal ellipse	Vertical ellipse
equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a > b$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1, a<b,$
eccentricity	$\frac{c}{a}$ $where\ c=\sqrt{a^2-b^2},$	$\frac{c}{a}$ $where\ c=\sqrt{a^2-b^2}$
Foci	$(\pm ae, 0)$ or $(\pm c, 0)$ (lie on x axis)	$(0, \pm ae)$ or $(0, \pm c)$ (lie on y axis)
vertices	$(\pm a, 0)$ (lie on x axis)	$(0, \pm a)$ (lie on y axis)
Major axis	2a (major axis along x axis)	2a(major axis along y axis)
Minor axis	2b (minor axis along y axis)	2b (minor axis along x axis)
directrices	$x = \pm\frac{a}{e}$ (parallel to y axis)	$y = \pm\frac{a}{e}$ (parallel to x axis)
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

- parameterization of ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ becomes $x=a\cos t$, $y=b\sin t$

5. A hyperbola is the locus of a point which moves such that, ratio of its distance from a fixed point (focus) and its distance from a fixed straight line (directrix), is a constant (eccentricity). This constant (eccentricity) is greater than unity.

- Hyperbola formula:

Nature of Hyperbola	Horizontal Hyperbola	Vertical Hyperbola
equation	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1,$	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1,$
eccentricity	$\frac{c}{a}$ $where\ c=\sqrt{a^2+b^2},$	$\frac{c}{a}$ $where\ c=\sqrt{a^2+b^2}$
Foci	$(\pm ae, 0)$ or $(\pm c, 0)$ (lie on x axis)	$(0, \pm ae)$ or $(0, \pm c)$ (lie on y axis)
vertices	$(\pm a, 0)$ (lie on x axis)	$(0, \pm a)$ (lie on y axis)
Transvers axis	2a (along x axis)	2a(along y axis)
conjugate axis	2b (along y axis)	2b (along x axis)
directrices	$x = \pm\frac{a}{e}$ (parallel to y axis)	$y = \pm\frac{a}{e}$ (parallel to x axis)
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

- Any point on the hyperbola, in parametric form is $x = a\sec\theta$, $y = b\tan\theta$.

- RECTANGULAR HYPERBOLA - The equation of the rectangular hyperbola referred to its transverse and conjugate axes as coordinate axes is therefore $x^2 - y^2 = a^2$.

CH 12. THREE DIMENSIONAL GEOMETRY

- Coordinate of a point P in space is defined as ordered triplets of real numbers (x, y, z) then
  x = the distance of the point from the YZ plane
  y = the distance of the point from the ZX plane
  z = the distance of the point from XY plane .
  - Three mutually perpendicular lines XOX', YOY'and ZOZ' called the x, y and z axes divides the space into eight boxes called octants.

Position of the point	**sign of (x, y,z)**
OCTANT I	(+, +,+)
OCTANT II	(- , +,+)
OCTANT III	(- , - ,+)
OCTANT IV	(+, - ,+)
OCTANT V	(+, +, -)
OCTANT VI	(- , +, -)
OCTANT VII	(- , - , -)
OCTANT VIII	(+, - ,-)

- Coordinate of origin is (0, 0, 0)
- Coordinate of a point lying on an axis
  Any point on x axis is given by (a , 0, 0) - value of y and z coordinates are 0.
  Any point on y axis is given by (0, b,0) - value of x and z coordinates are 0.
  Any point on z axis is given by (0, 0,c)- value of x and y coordinates are 0.
- Coordinate of a point lying on a plane
  Any point on XY plane is given by (a, b, 0) - value of z coordinates is 0.
  Any point on YZ plane is given by (0, b, c) - value of x coordinates is 0.
  Any point on ZX plane is given by (a, 0, c) - value of y coordinates is 0.
- The distance between points $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$ is defined by the formula
  $P_1P_2 = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2}$.
- Distance of the point P(x,y,z) from the origin is $\sqrt{x^2 + y^2 + z^2}$.
- Section formula - the coordinate of a point R dividing the line segment PQ joining P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) in the ratio m : n is given by

<u>Internal division</u>

$$x = \frac{mx_2+nx_1}{m+n},\ y = \frac{my_2+ny_1}{m+n},\ z = \frac{mz_2+nz_1}{m+n}$$

<u>External division</u>

$x = \frac{mx_2 - nx_1}{m-n}$, $y = \frac{my_2 - ny_1}{m-n}$, $z = \frac{mz_2 - nz_1}{m-n}$

- Midpoint formula: $x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$, $z = \frac{z_1 + z_2}{2}$
- Coordinate of centroid of a triangle with vertices A $(x_1, y_1, z_1), B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$is given by $x = \frac{x_1 + x_2 + x_3}{3}$, $y = \frac{y_1 + y_2 + y_3}{3}$, $z = \frac{z_1 + z_2 + z_3}{3}$
- Coordinate of centroid of a tetrahedron with vertices A $(x_1, y_1, z_1), B(x_2, y_2, z_2), C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$ is given by $x = \frac{x_1 + x_2 + x_3 + x_4}{4}$, $y = \frac{y_1 + y_2 + y_3 + y_4}{4}$, $z = \frac{z_1 + z_2 + z_3 + z_4}{4}$

CH 13. LIMITS AND DERIVATIVES

I.Formulas Of Limits

a. Change of base rule for logs: $\log_a x = \frac{\ln x}{\ln a}$

b. $\lim_{x \to 0} \frac{\sin x}{x} = 1$

c. $\lim_{x \to \infty} \frac{\sin x}{x} = 0$

d. $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

e. $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

f. $\lim_{x \to 0} \frac{a^x - 1}{x} = log_e a$

g. $\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$

II. DIFFERENTIATION

1.**Definition of derivative** : If y = f(x) then y'= y_1 = $\frac{df(x)}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

- A function f of x is differentiable if it is continuous.
- **Left hand derivative** – LHD = Lf'(a) = $\lim_{x \to a^-} \frac{\boldsymbol{f(a-h) - f(a)}}{\boldsymbol{-h}}$
- **Right hand derivative** – RHD = R f'(a) = $\lim_{x \to a^+} \frac{f(a+h) - f(a)}{h}$
- When LHD & RHD both exist and are equal then f(x) is said to be derivable or differentiable.

2.**FORMULAS OF DERIVATIVES**

1. $\frac{d(C)}{dx} = 0$
2. $\frac{d(x)}{dx} = 1$
3. $\frac{d(x^n)}{dx} = nx^{n-1}$
4. $\frac{d(e^x)}{dx} = e^x$
5. $\frac{d(e^{ax+b})}{dx} = ae^{ax+b}$
6. $\frac{d(a^x)}{dx} = a^x . loga$
7. $\frac{d(logx)}{dx} = \frac{1}{x}$
8. $\frac{d(sinx)}{dx} = cosx$

9. $\frac{d(cosx)}{dx} = -sinx$
10. $\frac{d(tanx)}{dx} = sec^2x$
11. $\frac{d(cotx)}{dx} = -cosec^2x$
12. $\frac{d(secx)}{dx} = secxtanx$
13. $\frac{d(cosecx)}{dx} = -cosecxcotx$
14. $\frac{df(ax+b)}{dx} = af'(ax + \text{b})$

3.**RULES OF DIFFERENTIATION**

- **Chain rule** : if y = f(u) and u = g(x) then $\frac{dy}{dx} = \frac{df}{du}.\frac{du}{dx}$
- **Product rule** : If u and v are two functions of x then $\frac{d(u.v)}{dx} = u.\frac{dv}{dx} + v.\frac{du}{dx} = uv' + u'v$
- **Quotient rule** :If u and v are two functions of x then $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$

CH 14. MATHEMATICAL LOGIC

- A **statement** is a sentence that is either true or false.
- A **closed sentence** is an objective statement which is either true or false.
- An **open sentence** is a statement which contains a variable and becomes either true or false depending on the value that replaces the variable.
- A **compound statement** is a sentence that consists of two or more statements separated by logical connectors.
- The **negation** of statement p is "not p." The negation of p is symbolized by "~p." The truth value of ~p is the opposite of the truth value of p.
- A **conjunction** is a compound statement formed by joining two statements with the connector AND. The conjunction "p and q" is symbolized by $p \wedge q$. A conjunction is true when both of its combined parts are true; otherwise it is false.
- A **disjunction** is a compound statement formed by joining two statements with the connector OR. The disjunction "p or q" is symbolized by $p \vee q$. A disjunction is false if and only if both statements are false; otherwise it is true. The truth values of $p \vee q$ are listed in the truth table below.
- A **conditional statement**, symbolized by $p \rightarrow q$, is an if-then statement in which p is a hypothesis and q is a conclusion. The logical connector in a conditional statement is denoted by the symbol $\rightarrow$. The conditional is defined to be true unless a true hypothesis leads to a false conclusion. A truth table for $p \rightarrow q$ is shown below.
- A biconditional statement is defined to be true whenever both parts have the same truth value. The biconditional operator is denoted by a double-headed arrow $\leftrightarrow$. The biconditional $p \leftrightarrow q$ represents "p if and only if q," where p is a hypothesis and q is a conclusion. The following is a truth table for biconditional $p \leftrightarrow q$.
- $p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$
- A **tautology** is always true, no matter what the truth-values of its component statements (if any) are.
- A **contradiction** is always false, no matter what the truth-values of its component statements (if any) are.
- When two statements have the same exact truth values, they are said to be logically equivalent. The biconditional of two equivalent statements is a tautology.

- **TRUTH TABLE** A **truth table** helps us find all possible truth values of a statement. Each statement is either True (T) or False (F), but not both.

p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

CH 15. STATISTICS

I. MEASURES OF CENTRAL TENDENCIES : The **mean** is the **average value** of the distribution.The arithmetic mean is the value obtained by adding all the data and dividing the result by the total number of data.

If $\bar{x}$ *is the A.M. of n observations* $x_1, x_2, x_3, \dots, x_n$ then $\bar{x} = \frac{x_1+x_2+\cdots+x_n}{n}$.

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = A + \left[\frac{1}{N}\sum_{I=1}^{n} f_i u_i\right] \times h \text{ , where } N = \sum_{i=1}^{n} f_i$$

II. Properties of the Arithmetic Mean

a. The sum of the deviations of all values of a distribution from their arithmetic mean is zero.

$\sum(X_i - \bar{X}) = 0$

b. The sum of the squares of the deviations of the values of the variable with respect to any number is minimized when the number matches the arithmetic mean.

$\sum(X_i - \bar{X})^2$ *Minimum*

c. If all values of the variable are added by the same number, the arithmetic mean is increased by that number.

d. If all values of the variable are multiplied by the same number, the arithmetic average is multiplied by that number.

III. Observations on the Arithmetic Mean

- The average can be found only in quantitative variables.
- The mean is independent of the widths of the classes.
- The mean is very sensitive to extreme scores.
- The mean cannot be calculated if there is a class with an indeterminate width.

IV. **MEDIAN** is the value of the variable which divides the distribution in two equal parts.

FOR INDIVIDUAL DATA – Arrange the data in ascending order. Count the total no. of observations (n).

- If total no. of observation n is odd then median is the value of $\left(\frac{n+1}{2}\right)^{th}$ observation.

- If total no. of observation n is even then median is the mean of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observation.

FOR GROUPED DATA

- First make the column of cumulative frequency(cf) in the frequency distribution table
- Decide the median class as the class with cumulative frequency $\geq \frac{N}{2}$
- Calculate median by using the formula **median =** $\boldsymbol{l + \frac{\frac{N}{2}-cf}{f} \times h}$, where

l = lower class limit of the median class,
cf = the cumulative frequency of the class preceding the median class
f = frequency of the median class
h = the class size.
N = total no. of observations

V. MODE is that value of the variable which has maximum frequency.

MODE OF A CONTINUOUS OR GROUPED FREQUENCY DISTRIBUTION

- First decide the modal class as the class corresponding to maximum frequency.
- Calculate mode by using the formula **mode =** $\boldsymbol{l + \left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right) \times h,}$ where

l = lower class limit of the medal class,
f_0 = the frequency of the class preceding the modal class
f_1 = frequency of the modal class
f_2 = the frequency of the class succeeding the modal class
h = the class size.

The three measures of central tendencies are connected by the relation :

Mode = 3 median – 2 mean

VI. MEASURES OF DISPERSION

Dispersion means scatterdness. The degree to which numerical data tend to spread about an average value is called the dispersion of the data. There are four measures of dispersion.

I. Range: Range = L – S, where L = largest value; S = smallest value.
Coefficient of Range = L - S / L + S

II. Mean Deviation:

It is the average of the modulus of the deviations of the observations in a series taken form mean or median.

Methods for Calculation of Mean Deviation:

Case I: For Ungrouped Data. In this case the mean deviation is given by the formula
Mean Deviation = M.D. = Σ|x - A|/n = Σ|d|/n,
Where ‘d’ stands for the deviation from the mean or median and |d| is always positive whether d itself is positive or negative and n is the total number of items.

Case II: For Grouped data.
Let $x_1, x_2, x_3, \ldots, x_n$ occur with frequencies $f_1, f_2, f_3, ,f_n$ respectively and let Σf = n and M can be either Mean or Median, then the mean deviation is given by the formula.
Mean Deviation = Σf|x-M|/Σf = Σf|d|/n
Where d = |x – M| and Σf = n.
Coefficient of Mean Deviation = Mean Deviation / Median
or = Mean Deviation / Mean (In case the deviations are taken from mean)

III. **Variance:** The average of the **squared** differences from the Mean. The square of the standard deviation is called variance and is denoted by σ^2.

a. For raw data $\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{X})^2$

b. For discreet data or grouped data: $\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} f_i(x_i - \bar{X})^2$ where $N = \sum_{i=1}^{n} f_i$

Or $\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i)^2 - (\bar{X})^2 = \frac{1}{N}\left(N\sum_{i=1}^{N}(x_i)^2 - (\bar{X})^2\right) = \frac{h^2}{N^2}\left(N\sum_{i=1}^{N} f_i {u_i}^2 - \{\sum_{i=1}^{n}(f_i u_i)\}^2\right)$ where $u_i = \frac{x_i - A}{h}$, A = assumed mean, h = class size

IV. Properties of the Variance

1 The variance is always positive or in the event that the values are equal, the variance is zero.
2 If all values of the variable are added by the same number, the variance does not change.
3 If all values of the variable are multiplied by the same number, the variance is multiplied by the square of that number.
4 If there are multiple distributions with the same mean and their variances are known, the total variance can be calculated.

If all samples have the same size: $$\sigma^2 = \frac{\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2}{n}$$

If the samples have different size: $$\sigma^2 = \frac{k_1 \cdot \sigma_1^2 + k_2 \cdot \sigma_2^2 + \ldots + k_n \cdot \sigma_n^2}{k_1 + k_2 + \ldots + k_n}$$

Observations on the Variance
1 The variance, like the average, is an index sensitive to extreme scores.
2 In cases where the mean cannot be found, it will not be possible to find the variance.
3 The variance is not expressed in the same units as the data since the deviations are squared.

V. Standard Deviation:

The Positive square root of the VARIANCE is called standard deviation. It is generally denoted by the Greek alphabet σ .
The formula for Standard Deviation:

a. For raw data $\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{X})^2}$

b. For discrete data or grouped data: $\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N} f_i(x_i - \bar{X})^2}$ where N = $\sum_{i=1}^{n} f_i$

Or $\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i)^2 - (\bar{X})^2} = \frac{1}{N}\sqrt{N\sum_{i=1}^{N}(x_i)^2 - (\bar{X})^2} = \frac{h}{N}\sqrt{N\sum_{i=1}^{N} f_i {u_i}^2 - \{\sum_{i=1}^{n}(f_i u_i)\}^2}$, where $u_i = \frac{x_i - A}{h}$, A = assumed mean, h = class size

VI. The **coefficient of variation** is the **ratio** between the **standard deviation** of a sample and its **mean**.$C.V. = \frac{\sigma}{\bar{X}} \times 100$
It allows us to compare the dispersions of two different distributions if their means are positive. The coefficient of variation for a distribution can be calucated to compare the values obtained with another distribution. The greater dispersion corresponds to the value of the coefficient of greater variation.

CH 16. PROBABILITY THEORY

- An **experiment** is a situation involving chance or probability that leads to results called outcomes.
- An **outcome** is the result of a single trial of an experiment.
- An **event** is one or more outcomes of an experiment.
- The **sample space** of an experiment is the set of all possible outcomes of that experiment.
- **Probability** is the measure of how likely an event is.
- The probability of event A is the number of ways event A can occur divided by the total number of possible outcomes P(A) = $\frac{No.of\ outcomes\ favourable\ to\ the\ event}{Total\ no\ of\ outcomes}$

- If P(A) > P(B) then event A is more likely to occur than event B.
- If P(A) = P(B) then events A and B are equally likely to occur.
- If event A is impossible, then P(A) = 0.
- If event A is certain, then P(A) = 1.
- The complement of event A is $\bar{A}$. P($\bar{A}$) = 1 - P(A)
- The probability of a sample point ranges from 0 to 1.
- The sum of the probabilities of the distinct outcomes within a sample space is 1.
- Two events are **mutually exclusive** if they cannot occur at the same time (i.e., they have no outcomes in common).
- Two events, A and B, are **independent** if the fact that A occurs does not affect the probability of B occurring.
- Two events are **dependent** if the outcome or occurrence of the first affects the outcome or occurrence of the second so that the probability is changed.
- **Addition Rule1:** When two events, A and B, are mutually exclusive, the probability that A or B will occur is the sum of the probability of each event. P(A or B) = P(A) + P(B)
- **Addition Rule2:** When two events, A and B, are non-mutually exclusive P(A or B) = P(A) + P(B) - P(A and B)
- **Addition Rule3:** When two events, A and B, are independent, the probability that A or B will occur is P(A or B) = P(A) + P(B) - P(A) · P(B)
- **Multiplication Rule 1 :**When two events, A and B, are independent, the probability of both occurring is: P(A and B) = P(A) · P(B)