Points to Remember

In Mathematics

Class – X

BY :

Balraj Khurana

Index **PAGE NO.**

CH.1 - REAL NUMBERS

- **Euclid's Division Lemma:** Given any two positive integers a and b (a>b), there exists unique integers q and r such that a = bq + r, $0 \leq r < b$.
- **Fundamental Theorem of Arithmetic**(**Unique Factorisation Theorem**): Every composite number can be uniquely expressed as a product of prime numbers
- To find **HCF** of two given numbers a and b

I. By using **Euclid's Division Lemma**:
   I. Apply Euclid's Division Lemma to a and b to obtain q and r such that a = bq + r, $0 \leq r < b$.
   II. If r = 0, b is the HCF of a and b. If r$\neq 0$, apply the lemma to b and r.
   III. Continue the process till you get the remainder 0. The divisor at this stage is the required HCF.

II. By using **Fundamental Theorem of Arithmetic**
   I. Find the prime factorisation of each number.
   II. For HCF consider product of common factors only.
   III. For LCM take product of all the factors, taking the product of common factors only once.

- If a prime number p divides a^2, then p divides a also.
- If p is a prime number then $\sqrt{p}$ is an irrational number.
- A rational number $\frac{p}{q}$ will have a terminating decimal representation if q is of the form $2^m \times 5^n$, where m and n are non-negative integers.

CH.2 - POLYNOMIALS

- An algebraic expression of the form $p(x)= a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_nx^n$ is called a **POLYNOMIAL** in the variable x, where $a_0, a_1, a_2, ..., a_n$ are real numbers ($a_n \neq 0$)called the coefficients of the polynomial and n is a non negative integer.
- An algebraic expressions in which the exponents (powers) of the variable are negative or fractions is <u>NOT</u> a polynomial.
- The greatest exponent of the variable is called the **<u>DEGREE</u>** of the polynomial.
- A value of the variable which makes the value of the polynomial zero is called the **<u>ZERO</u>** of the polynomial.
- A polynomial of degree n has n zeroes.
- Graphically the zeroes of a polynomial are the x coordinates of the point of intersection of the graph of y = p(x) with the x axis.
- If α, β *are the zeroes of a polynomial* $\boldsymbol{ax^2} + \boldsymbol{bx} + \boldsymbol{c}, a \neq 0,$ *then* $\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$.
- If the relation between zeroes of a polynomial are given ,then the polynomial can be constructed as : **<u>x^2 – (sum of the zeroes)x + (product of the zeroes).</u>**
- If α, β *and* γ *are the zeroes of a cubic polynomial* $\boldsymbol{ax^3} + \boldsymbol{bx^2} + \boldsymbol{cx} + \boldsymbol{d}, a \neq 0,$ *then*
  $\alpha + \beta + \gamma = -\frac{b}{a}, \quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}, \quad \alpha\beta\gamma = -\frac{d}{a}$.
- If the relation between zeroes of a cubic polynomial are given ,then the polynomial can be constructed as: **<u>x^3 – (sum of the zeroes)x^2 + (pairwise product of the zeroes)x + (product of the zeroes).</u>**
- **Division algorithm**: When a polynomial p(x) is divided by another non-zero polynomial g(x), then there exists two polynomials q(x) and r(x) such that **<u>p(x)= g(x).q(x) + r(x)</u>** , where r(x)=0 or degree of r(x)<

degree of g(x). p(x), g(x), q(x) and r(x) are respectively called the dividend, divisor , quotient and remainder polynomials.

CH.3-A SYSTEM OF LINEAR EQUATIONS IN TWO VARIABLES

$a_1x + b_1y + c_1 = 0$; A common solution to the system of linear equations is a set of values
$a_2x + b_2y + c_2 = 0$. of x and y for which both the equations are satisfied.

a_1, a_2 are the coefficients of x, b_1, b_2 are the coefficients of y and c_1, c_2 are the constant terms.

- **Graphical method** of solving the system of linear equations
  - I. Find the SOLUTION SET for each equation by expressing one variable in term of the other and taking suitable values for independent variable and obtaining the corresponding value of the dependent variable.
  - II. **Plot** the points corresponding to the solutions of one equation, **join** the points by a straight line and **extend** the line on both the sides , **put** arrow heads and **write** the equation of the line on it.
  - III. Repeat the same for the second equation.
  - IV. If both the lines are intersecting ,the system has a UNIQUE SOLUTION given by the coordinates of the point of intersection .
  - V. If both the lines are parallel,the system has a NO SOLUTION.
  - VI. If both the lines are coincident ,the system has a INFINITE SOLUTION.
- ELIMINATION BY **SUBSTITUTION METHOD**
  - I. Express one variable in term of the other from one equation and substitute this value in the second equation.
  - II. The second equation reduces to an equation in single variable, solve it.
  - III. Put the obtained value of the variable in the expression for second variable.
  - IV. The values obtained of both the variables are the required solution of the system.
- ELIMINATION BY **EQUATING THE COEFFICIENTS METHOD**
  - I. Equate the coefficients of one variable by multiplying the equations by suitable numbers.
  - II. If the equal coefficients are opposite in signs then we eliminate the variable by adding the equations.
  - III. If the equal coefficients are same in signs then we eliminate the variable by subtracting the equations.
  - IV. We obtain the value of the remaining variable
  - V. Put this value in any of the equations to get the value of the other variable.
- **CROSS MULTIPLICATION METHOD** –
  - I. List down values of a_1, a_2, b_1, b_2, c_1 and c_2 from the given equations.
  - II. Use the format $\dfrac{x}{\begin{matrix} b_1 & c_1 \\ b_2 & c_2 \end{matrix}} = \dfrac{y}{\begin{matrix} c_1 & a_1 \\ c_2 & a_2 \end{matrix}} = \dfrac{1}{\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \end{matrix}}$ for the equations $a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$

    Use the format $\dfrac{x}{\begin{matrix} b_1 & c_1 \\ b_2 & c_2 \end{matrix}} = \dfrac{y}{\begin{matrix} c_1 & a_1 \\ c_2 & a_2 \end{matrix}} = \dfrac{-1}{\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \end{matrix}}$ for the equations $a_1x + b_1y = c_1$; $a_2x + b_2y = c_2$
  - III. By cross multiplication write x = $\dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and y = $\dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

CH4 – QUDRATIC EQUATION

$ax^2 + bx + c = 0$, $a \neq 0$, is a quadratic equation in x where a is the coefficient of x^2,b is the coefficient of x and c is the term independent of x.

Solutions of the equation are those values of x which will satisfy the equation .

I. Solving the quadratic equation by **factorization method**

- Express $ax^2 + bx + c = 0$ as $(x-p)(x-q) = 0$, where $-(p+q) = b$ and $pq = ac$.
- If product ac is positive then p and q has same sign.
- If product ac is negative then p and q has opposite signs.
- Smaller of p or q will have the same sign as b.
  Then x = p and q will be the required solutions.

II. Solving the quadratic equation by **completion of square method**

- Divide $ax^2 + bx + c = 0$ by a i.e. coefficient of x^2 to get $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
- Add $(\frac{b}{2a})^2$ i.e. (1/2 of coefficient of $x)^2$on both sides to get $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$
- Express LHS as a perfect square and take lcm on RHS $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$
- Take square root of both the sides to obtain $x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$
- Express x as $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$
- Obtain the required solution as $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$, this is known as the **QUADRATIC FORMULA or THE SRIDHARACHARYA'S FORMULA.**

III. **Discriminant** of a quadratic equation: $D = b^2 - 4ac$, is a number attached to every quadratic equation that decides the nature of solution inyhe following way:

- $D > 0$ - roots of the equation are REAL AND DISTINCT.
- $D = 0$ - roots of the equation are REAL AND EQUAL, i.e. repeated roots.
- $D < 0$ – NO REAL ROOTS

CH 5.- ARITHMETIC PROGRESSION

A sequence of numbers is in ARITHMETIC PROGRESSION if the difference between consecutive terms is always the same.

- If **a** is the 1st term and **d** is the common difference, A.P. can be written as **a, a+d, a+2d, a+3d, ...**
- A sequence a_1 , a_2, a_3, ..., a_n is an A.P. if a_n is a linear polynomial in n, i.e. $a_n = pn + q$, then the coefficient of n in the linear polynomial becomes the common difference of the A.P.
  Different terms of the A.P. can be obtained by putting n = 1, 2, 3,... in the expression for a_n.
- Formula for n^{th} term - **$a_n = a + (n-1)d$** ,
- sum of first n terms of an A.P. – **$S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = \frac{n}{2}[a + l]$**
- **Methods to solve a problem**
  - List the given values from the question.
  - Note down what has to be calculated.
  - Write the suitable formula.

- There are four variables involved in the formula, out of which any three values will be given in the question.
- Then find the unknown by using the formula

- For two A.P.s with the same common difference, the difference between corresponding terms is the same as the difference between the first terms, i.e. $\mathbf{a_n - b_n = a - b}$
- **The n^{th} term from the end –** if there is an A.P. with first term a and common difference d and total number of terms m, then n^{th} term from the end = $(m – n +1)^{th}$ term from the beginning.
  Also, n^{th} term from the end = last term + (n – 1) (- d) = l – (n – 1)d.
- **Various terms of an A.P. can be chosen in the following way:**

No. Of terms	Terms	common difference
3	**a-d, a, a+d**	**d**
4	**a – 3d , a – d , a + d , a+ 3d**	**2d**
5	**a – 2d , a – d , a , a + d , a + 2d**	**d**

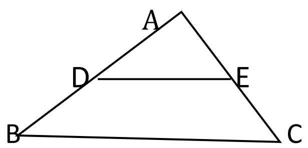
CH 6. – TRIANGLES

- **Basic Proportionality theorem or Thales theorem** – If a line is drawn parallel to one side of the triangle to divide the other two sides in distinct points then the other two sides are divided in the same ratio, i. e. If DE∥ $BC \ in \ \triangle ABC \ then \ \frac{AD}{BD} = \frac{AE}{EC}$
  Following results are equivalent to the result of BPT

i. $\frac{AD}{BD} = \frac{AE}{EC}$ iv. $\frac{BD}{AD} = \frac{EC}{AC}$

ii. $\frac{AD}{AB} = \frac{AE}{AC}$ v. $\frac{AB}{AD} = \frac{AC}{AE}$

iii. $\frac{BD}{AB} = \frac{EC}{AC}$ vi. $\frac{AB}{BD} = \frac{AC}{EC}$



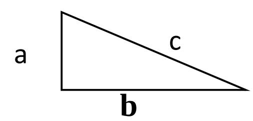
- Converse of **Basic Proportionality theorem –** if a line divides the two sides of a triangle in the same ratio then the line must be parallel to the third side.
- In solving the questions if a line parallel to one side is given , apply BPT
- IF sides are divided by a line in equal ratio is given , apply the converse of BPT

Similar Triangles - Two triangles are similar if their corresponding angles are congruent and their corresponding sides are proportional.

- **AAA Test:** If all angles of one triangle are congruent with the corresponding two angles of another triangle, the two triangles are similar.
- **A - A Test:** If two angles of one triangle are congruent with the corresponding two angles of another triangle, the two triangles are similar.The sum of all three angles of a triangle is 1800. Therefore if two angles are congruent the third is automatically congruent. Therefore the sufficient condition requires only two angles to be congruent.
- **SAS Test:** If two sides of one triangle are proportionate to the two corresponding sides of the second triangle and the angles between the two sides of each triangle are equal the two triangles are similar.
- **SSS Test:** If the three sides of one triangle are proportional to the three corresponding sides of another triangles, then two triangles are similar.

Properties of Similar triangle

- In two triangles if the corresponding angles are congruent, their corresponding sides are proportional.
- If the sides of two triangles are proportional then the corresponding angles are congruent.
- **Perimeters of similar triangles:** Perimeters of similar triangles are in the same ratio as their corresponding sides and this ratio is called the **SCALE FACTOR**.i.e. If Δ ABC ~ Δ PQR, then $\frac{\text{perimeter } \Delta \text{ ABC}}{\text{perimeter } \Delta \text{ PQR}} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$
- **Areas of similar triangles:** The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides, i.e. the square of the scale factor. If Δ ABC ~ Δ PQR, then $\frac{\text{Area } \Delta \text{ ABC}}{\text{Area } \Delta \text{ PQR}} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$
- **Pythagoras' Theorem -** In a right angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides i.e. $a^2+b^2 = c^2$

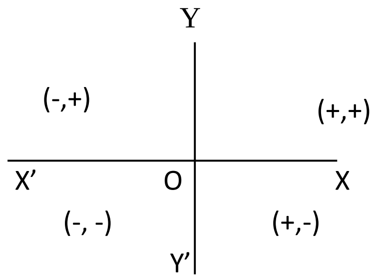


- **Converse of Pythagoras'** Theorem - If the square on one side of a triangle is equal to the sum of the squares on the other two sides, the angle contained by these two sides is a right angle. In⊿ABC, if $BC^2 = AB^2 + AC^2$ then, ∠A is a right angle.

CH 7- COORDINATE GEOMETRY

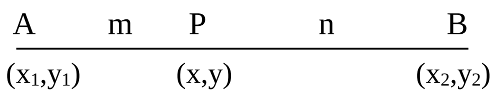
- Coordinate of a point P in a plane is denoted by the ordered pair (x,y), then
  x = abscissa = the distance of the point from the y axis
  y = ordinate = the distance of the point from the x axis
- **Position of the point** / **sign of (x, y)**

Position of the point	sign of (x, y)
Quadrant I	(+, +)
Quadrant II	(- , +)
Quadrant III	(- , -)
Quadrant IV	(+, -)



- Any point on x axis is given by (a , 0)- value of y coordinate is 0.
  Any point on y axis is given by (0, b)- value of x coordinate is 0.
- **DISTANCE FORMULA-** The distance between the two points $P(x_1,y_1)$ and $Q (x_2,y_2)$ is given by $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- d = $\sqrt{(\textbf{difference of abscissa})^2 + (\textbf{difference of ordinate})^2}$.
  The order in which the points P and Q has been taken as the first point or the second point is not important as it does not affect the formula.
- Distance of a point P (a,b) from the origin is OP = $\sqrt{a^2 + b^2}$.
- **SECTION FORMULA -** The coordinate of the point P(x,y) dividing the line segment joining the points $A(x_1,y_1)$ and $B (x_2,y_2)$ in the ratio m: n is given by

$x = \frac{mx_2 + nx_1}{m+n}$ $y = \frac{my_2 + ny_1}{m+n}$



- Coordinate of the **MID -POINT** of a line segment joining the points A(x_1,y_1) and B (x_2,y_2) is given by
  $x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$
- **CENTROID** of a triangle with vertices A(x_1,y_1) ,B (x_2,y_2) and C (x_3,y_3) is given by
  $\mathbf{x} = \frac{x_1 + x_2 + x_3}{3}$, $\mathbf{y} = \frac{y_1 + y_2 + y_3}{3}$
- **The AREA** of a triangle with vertices A(x_1,y_1) ,B (x_2,y_2) and C (x_3,y_3) is given by
  $\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$
- **For three points A, B and C to be collinear**

✓ Show, by using **distance formula** ,that the sum of the two distances is equal to the third distance , if the coordinates are known and collinearity has to be established.

✓ Show ,by using the condition for collinearity $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$,that the area of the triangle formed by these three points as vertices is 0

✓ Show ,by using the **section formula**, that one of the points divides the line segment joining the other two points in a fixed ratio

✓ Use the 2nd or 3rd method if three points are collinear given and unknown coordinate is to be calculated.

- In order to prove that the given points are vertices of

✓ **ISOSCELES TRIANGLE** – find the lengths of the sides by using distance formula and show that any two distances are equal, i.e. **AB= AC or AB = BC or BC = AC**

✓ **EQUILATERAL TRIANGLE** – find the lengths of the sides by using distance formula and show that all the distances are equal. i.e. **AB = BC = AC**

✓ **RIGHT ANGLED TRIANGLE** – find the lengths of the sides by using distance formula and show that they satisfy the Pythagoras formula. i.e. **$AB^2 = BC^2 + AC^2$ or $AC^2 = AB^2 + BC^2$ or $BC^2 = AB^2 + AC^2$**

✓ **PARALLELOGRAM** – show that the midpoints of the diagonals are the same since in a parallelogram the diagonals bisect each other. i.e.**mid -point of AC = mid -point of BD**
Or show that opposite sides are equal by using DISTANCE FORMULA.i.e. **AB = CD, BC = DA**

✓ **RHOMBUS** – find the lengths of the sides by using distance formula and show that all these distances are equal.i.e. **AB = BC =CD = DA**

✓ **RECTANGLE** – find the lengths of the sides and diagonals by using distance formula and show that opposite sides are equal and diagonals are equal. i.e.**AB = CD, BC = DA and AC=BD**

✓ **SQUARE** – find the lengths of the sides and diagonals by using distance formula and show that all the sides are equal and diagonals are also equal.i.e. **AB = BC =CD = DA and AC=BD**

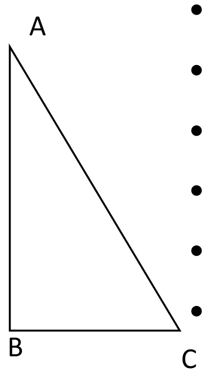
✓ **PARALLELOGRAM BUT NOT A RECTANGLE -** find the lengths of the sides and diagonals by using distance formula and show that opposite sides are equal and diagonals are not equal.i.e. **AB = CD, BC = DA and AC ≠ BD**

✓ **RHOMBUS BUT NOT A SQUARE** – find the lengths of the sides and diagonals by using distance formula and show that all the sides are equal and diagonals are not equal.i.e. **AB = BC =CD = DA and AC ≠ BD**

The order in which vertices are given in the question is important so do not change the order, take the vertices in cyclic order.

CH 8 – INTRODUCTION TO TRIGONOMETRIC RATIOS

TRIGONOMETRIC RATIOS - In a right triangle ABC the trigonometric ratios are defined as follows:



- **SINE** of angle C= sinC = $\frac{side\ opposite\ to\ the\ angle}{hypotenuse} = \frac{AB}{AC} = \frac{p}{h}$
- **COSINE** of angle C= cosC = $\frac{side\ adjacent\ to\ the\ angle}{hypotenuse} = \frac{BC}{AC} = \frac{b}{h}$
- **TANGENT** of angle C= tanC = $\frac{side\ opposite\ to\ the\ angle}{side\ adjacent\ to\ the\ angle} = \frac{AB}{BC} = \frac{p}{b} = \frac{sinC}{cosC}$
- **COSECANT** of angle C = cosecC = $\frac{hypotenuse}{side\ opposite\ to\ the\ angle} = \frac{AC}{AB} = \frac{h}{p} = \frac{1}{sinC}$
- **SECANT** of angle C= secC = $\frac{hypotenuse}{side\ adjacent\ to\ the\ angle} = \frac{AC}{BC} = \frac{h}{b} = \frac{1}{cosC}$
- **COTANGENT** of angle C = cotC = $\frac{side\ adjacent\ to\ the\ angle}{side\ opposite\ to\ the\ angle} = \frac{BC}{AB} = \frac{b}{p} = \frac{1}{tanC} = \frac{cosC}{sinC}$

USEFUL IDENTITIES – $\sin^2A + \cos^2A = 1$ $\sin^2A = 1 - \cos^2A$ $\cos^2A = 1 - \text{Sin}^2A$

$\sec^2A - \tan^2A = 1$ $\sec^2A = 1 + \tan^2A$ $\tan^2A = \sec^2A - 1$

$\text{cosec}^2A - \cot^2A = 1$ $\text{cosec}^2A = 1 + \cot^2A$ $\cot^2A = \text{cosec}^2A - 1$

TRIGONOMETRIC RATIOS OF COMPLEMENTARY ANGLES

- $\sin(90° - A) = cosA$
- $cos\ (90° - A) = \sin A$
- $\sec(90° - A) = cosecA$
- $cosec\ (90° - A) = \sec A$
- $\tan(90° - A) = cotA$
- $cot\ (90° - A) = \tan A$

TRIGONOMETRIC RATIOS OF SPECIFIC ANGLES

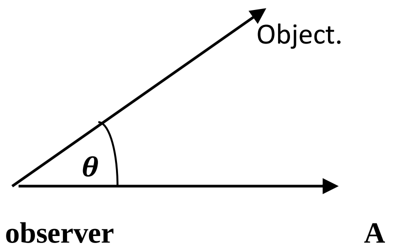
	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

CONVERSION FORMULA for expressing one trigonometric ratio in term of other

	sinA	cosA	tanA	cosecA	secA	cotA
sinA	sinA	$\sqrt{1-sin^2A}$	$\frac{sinA}{\sqrt{1-sin^2A}}$	$\frac{1}{sinA}$	$\frac{1}{\sqrt{1-sin^2A}}$	$\frac{\sqrt{1-sin^2A}}{sinA}$
cosA	$\sqrt{1-cos^2A}$	cosA	$\frac{\sqrt{1-cos^2A}}{cosA}$	$\frac{1}{\sqrt{1-cos^2A}}$	$\frac{1}{cosA}$	$\frac{cosA}{\sqrt{1-cos^2A}}$
tanA	$\frac{tanA}{\sqrt{1+tan^2A}}$	$\frac{1}{\sqrt{1+tan^2A}}$	tanA	$\frac{\sqrt{1+tan^2A}}{tanA}$	$\sqrt{1+tan^2A}$	$\frac{1}{cosA}$
cosec A	$\frac{1}{cosecA}$	$\frac{\sqrt{cosec^2A-1}}{cosecA}$	$\frac{1}{\sqrt{cosec^2A-1}}$	cosecA	$\frac{cosecA}{\sqrt{cosec^2A-1}}$	$\sqrt{cosec^2A-1}$
secA	$\frac{\sqrt{sec^2A-1}}{secA}$	$\frac{1}{secA}$	$\sqrt{sec^2A-1}$	$\frac{secA}{\sqrt{sec^2A-1}}$	secA	$\frac{1}{\sqrt{sec^2A-1}}$
cotA	$\frac{1}{\sqrt{1+cot^2A}}$	$\frac{cotA}{\sqrt{1+cot^2A}}$	$\frac{1}{cotA}$	$\sqrt{1+cot^2A}$	$\frac{\sqrt{1+cot^2A}}{cotA}$	cotA

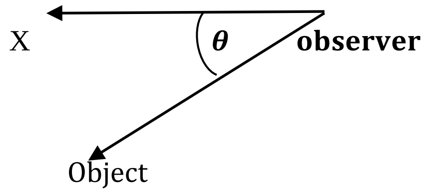
<u>CH 9 – HEIGHTS AND DISTANCES</u>

Line of sight - the line joining the observer's eye and the object



Angle of elevation – when the object is above the eye level , the angle between the horizontal line through the observer's eye and the line of sight is called the angle of elevation.

Angle of elevation – when the object is below the eye level, the angle between the horizontal line through the observer's eye and the line of sight is called the angle of elevation.



Steps for solving a question

- Read the problem carefully, draw the figure neatly and list down the given quantities.
- Explain the figure .
- According to the given facts decide which trigonometric ratio to consider and in which triangle.
- Formulate the equations by considering one right triangle at a time and simultaneously solve both the equations.

Points to remember –

- As we move closer to an object the angle of elevation or depression increases in measure.
- The object and its image are at equal height and depth from the horizontal level(water level).

- Two angles are complementary $\Rightarrow \alpha + \beta = 90°$.
- Take $\sqrt{2} = 1.41 \ and \ \sqrt{3} = 1.73$

CH10- CIRCLES

Definitions

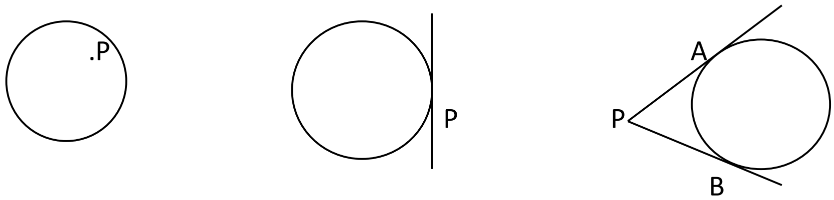
- **Circle –** The locus of a moving point such that its distance from a fixed point is always fixed**.** The fixed point is called the **CENTRE** and the fixed distance is called the **RADIUS.**
- **Radius** – A line segment joining the centre to any point on the circle
- **Chord** – a line segment joining two points on the circle.
- **Diameter**- A chord passing through the centre of the circle
- **Minor arc** - An arc measuring less than 180°
- **Semicircle** - An arc whose end points lie on a diameter of a circle
- **Major arc** - An arc measuring more than 180°
- **Inscribed angle -** An angle whose vertex lies on the circle
- **Concyclic points** - Points which lie on the circumference of the same circle .
- A **cyclic quadrilateral** is a quadrilateral with all its four corners (vertices) on the circumference of the same circle.
- **Secant** - A line that intersects a circle in two separate points.
- **Tangent** - A line that intersects a circle in only one point
- **Point of tangency** - The point where a tangent intersects a circle.
- **Common internal tangent**- A line that is tangent to more than one circle and crosses in between the two circles.
- **Internally tangent circles** -Circles that intersect in only one point with one circle inside of the other (sharing interior points)
- **Common external tangent -** A line that is tangent to more than one circle but does not cross between the circles
- **Externally tangent circles** - Circles that intersect in only one point and have no other points in common

Properties of circles

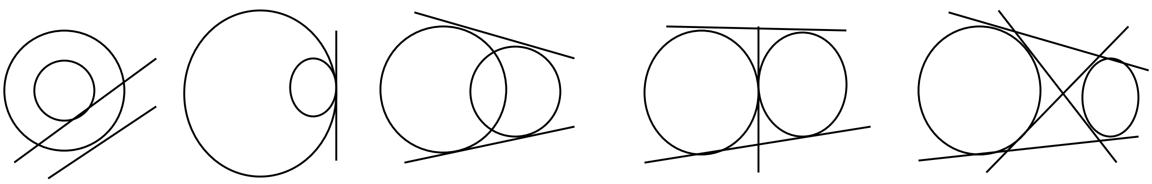
- Two circles are congruent if their radii are equal.
- Two circles are concentric if they have the same centre.
- Two arcs are congruent if their degree measure are equal.
- Two arcs are congruent if their corresponding chords are equal and vice versa.
- Perpendicular from centre bisects a chord
- Equal chords are equidistant from the centre.
- There is one and only one circle passing through three non-collinear points
- Angle subtended by a chord at the centre is twice the angle subtended at the circumference.
- Angles subtended by a chord in the same segment are equal.
- The angle in a semi-circle is a Right Angle.
- Opposite angles in a cyclic quadrilateral are supplementary.
- The exterior angle of a cyclic quadrilateral is equal to opposite interior angle.
- A tangent is perpendicular to the radius of a circle

- Two tangents drawn from an external point to the circle are equal. If two tangents are drawn from the point P to the circle C(O,r) to touch it at A and B, then
  - ✓ PA = PB
  - ✓ $\angle APO \cong \angle BPO$
  - ✓ $\Delta APO \cong \Delta BPO$
- The angle between a tangent and a chord is equal to any Angle in the alternate (opposite) segment.
- If two circles touch each other then the point of contact lies on the line joining the centres.

Position of an external point with respect to a circle	**number of tangents**
Point lying inside the circle	0
Point lying on the circle	1
Point lying outside the circle	2



Position of the circles	**number of common tangents**
If one circle lies completely inside the other circle	0
If the circles touch internally	1
If the two circles intersect at two points	2
If the two circles touch externally	3
If the two circles do not have any common points	4



Ch 11- Construction

SCALE FACTOR –The ratio of the sides of a triangle to be constructed similar to a given triangle .

I. **To draw the tangent to a circle at a point on it without using its centre**
   - Draw any chord through the given point.

- Draw an angle subtended by the chord on the major arc .
- Construct an angle of the same measure with the chord at the given point.
- The other arm of this angle is the required tangent.

II. To draw the tangent to a circle at a point on it by using its centre

- Join the centre with the given point
- Draw a perpendicular to this radius at the given point
- This is the required tangent .

III. To draw a pair of tangents to a circle from an external point

- Join the centre O with the point P
- Bisect OP
- With midpoint M of OP as centre and MP=MO as radius draw a circle to intersect the given circle at A and A'
- Join PA and PA' which are the required tangents.

IV. To draw a pair of tangents to a circle from an external point without using the centre

- Draw a secant PAB from the external point Pto intersect the circle at A and B.
- Produce AP to C such that AP = CP.
- Draw a semicircle with CB as the diameter.
- Draw PD⊥ CB, with P as centre and PD as radius draw arcs to intersect the given circle at T and T'
- Join PT and PT' to get the required tangents.

V. To divide a line segment in given ratio m:n

- Draw a line segment AB of given length by using a ruler.
- Draw a ray AX at A making an acute angle with AB.
- Draw a number of equal arcs (sum of the numerator and denominator of the ratio i.e. m+n) on AX at the points $A_1, A_2, \ldots, A_{m+n.}$
- Join A_{m+n} with B.
- From the arc point corresponding to the numerator of the given ratio i.e. A_m draw a line parallel to A_{m+n} B by drawing corresponding angles by using compass and ruler to meet AB at P.
- Then AP:BP = m:n.

VI. To draw a triangle similar to a given triangle with scale factor m:n

- Construct the given triangle ABC as per the facts given.
- Draw a ray BX at B making an acute angle with BC.
- Draw a number of equal arcs (sum of the numerator and denominator of the ratio i.e. m+n) on BX at the points $B_1, B_2, \ldots, B_{m+n.}$
- Join B_{m+n} with C.
- From the arc point corresponding to the numerator of the given ratio i.e. B_m draw a line parallel to B_{m+n} C by drawing corresponding angles by using compass and ruler to meet BC at C'.
- From C' draw a line parallel to CA by using compass and ruler to cut AB at A'.
- A'BC' $\sim ABC$ with scale factor m:n.

VII. To draw a quadrilateral similar to a given quadrilateral with scale factor m:n

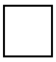
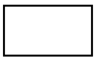
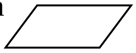
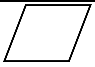
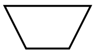
- Construct the given quadrilateral ABCD as per the facts given.

- Join AC.
- Draw a ray AX at A making an acute angle with BC.
- Draw a number of equal arcs (sum of the numerator and denominator of the ratio i.e. m+n) on AX at the points $A_1, A_2, \ldots, A_{m+n}$.
- Join A_{m+n} with B.
- From the arc point corresponding to the numerator of the given ratio i.e. A_m draw a line parallel to A_{m+n} B by drawing corresponding angles by using compass and ruler to meet AB at B'.
- From B' draw a line parallel to BC by using compass and ruler to cut AC at C'.
- From C' draw a line parallel to CD by using compass and ruler to cut AD at D'.
- AB'C'D' $\sim ABCD$ with scale factor m : n.

CH 12 – AREAS RELATED TO A CIRCLE

For a circle of radius r

- **Circumference = $2\pi r$**
- **Area of a circle** $A = \pi r^2$
- **Area of semicircle** $A = \frac{1}{2}\pi r^2$
- **Area of quadrant** $A = \frac{1}{4}\pi r^2$
- **Sector of a circle** – The part of the circular region enclosed between two radii ,OP and OQ , and the corresponding arc$\widehat{PQ}$ is called the sector OPQ. ∠POQ is called the angle of the sector.
  - ✓ **Minor sector** if arc PQ is minor arc.
  - ✓ **Major sector** if arc PQ is major arc.
  - ✓ Sum of the lengths of minor arc PQ and major arc PQ is equal to the circumference of the circle.
  - ✓ Sum of the areas of minor sector POQ and major sector POQ is equal to the area of the circle.
- If OPQ is a sector of a circle C(O,r) with angle θ and arc length l and area A, then
  - ✓ **Arc length of a sector** = l = $2\pi r \times \frac{\theta°}{360°}$ = $circumference\ of\ the\ circle \times \frac{\theta°}{360°}$.
  - ✓ **Area of a sector** = $A = \pi r^2 \times \frac{\theta°}{360°}$ = $area\ of\ the\ circle \times \frac{\theta°}{360°}$.
  - ✓ A = $\frac{1}{2} lr$.
  - ✓ **Perimeter** of a sector = l **+ 2r**
- **Segment of a circle** – The region enclosed between an arc and a chord is called a segment. Any chord PQ divides the whole circular region into two segments – minor segment and major segment
- **Area of ring or annulus (shaded region between two concentric circles of radii R and r)** = $\pi(R^2 - r^2)$
- If two circles touch each other internally , the distance between their centres is equal to the difference of their radii.
- If two circles touch each other externally , the distance between their centres is equal to the sum of their radii.
- Distance moved by rotating a wheel in one revolution is equal to the circumference of the wheel.
- Number of revolution completed in rotating a wheel in one minute = $\frac{\text{distance moved in one minute}}{\text{circumferrence}}$.
- To calculate the area of combination of plane figures
  - ✓ Draw the figure.
  - ✓ List down the dimensions given in the question.
  - ✓ Calculate required area = area of outer region – area of inner region
  - ✓ Use the formulae of mensuration for plane figures as given below:

FIGURE	DIMENSION	PERIMETER	AREA
EQUILATERAL TRIANGLE	Side = a	3a	$\frac{\sqrt{3}}{4}a^2$
Square 	Side = a	4a	a^2
rectangle 	Length= l breadth=b	2(l + b)	lb
parallelogram 	Base =b, height = h	2(l + b)	$\frac{1}{2}b \times h$
rhombus 	Side = a, d_1, d_2= length of the diagonals	4a	$\frac{1}{2}d_1 \times d_2$
trapezium 	b_1,b_2 = length of parallel sides h= height	Sum of the sides	$\frac{1}{2}(b_1+b_2) \times h$

CH 13- SURFACE AREAS AND VOLUMES

- Draw the figure and write down the dimensions according to the question.
- Determine what has to be calculated and apply formula accordingly.
- In problems based on conversion of solid (when one solid figure is melted and recasted into another solid figure) the volume of both the figures remain the same.
- In problems of combination of solid figures
  - ✓ Volume of combination = sum of the volumes of component solids
  - ✓ Surface area of the combination = sum of the curved surface areas of the component solids.
- In problems of frustum of a cone use the concept of similar triangles to determine the relation between the height and radius of smaller and bigger cone.
- Volume of the frustum = volume of bigger cone – volume of smaller cone.
- Important formulas

Figure	Dimension	Volume	Curved surface area	Total surface area
cone	Radius = r Height = h Slant height= l	$\frac{1}{3}\pi r^2 h$	πrl	$\pi r^2 + \pi rl$
cylinder	Radius = r Height = h	$\pi r^2 h$	$2\pi rh$	$2\pi r^2 + 2\pi rh$
sphere	Radius = r	$\frac{4}{3}\pi r^3$	$4\pi r^2$	$4\pi r^2$
hemisphere	Radius = r	$\frac{2}{3}\pi r^3$	$2\pi r^2$	$3\pi r^2$
Frustum of a cone	r_1,r_2 = radii of two circular bases h= height of the frustum	$\frac{1}{3}\pi h({r_1}^2 + r_1 r_2 + {r_2}^2)$	$\pi(r_1 + r_2)l$	$\pi(r_1 + r_2)l + \pi {r_1}^2 + \pi {r_2}^2$

CH 14 – STATISTICS

ARITHMETIC MEAN OF INDIVIDUAL OBSERVATIONS

If $\bar{x}$ *is the A.M. of n observations* $x_1, x_2, x_3, \dots, x_n$ then $\bar{x} = \frac{x_1+x_2+\cdots+x_n}{n}$.

ARITHMETIC MEAN OF DESCRETE FREQUENCY DISTRIBUTION

I. **DIRECT METHOD**
if $\bar{x}$ *is the A.M. of n observations* $x_1, x_2, x_3, \dots, x_n$ *with frequencies* $f_1, f_2, f_3, \dots, f_n$ *respectively, then*

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{f_1x_1+f_2x_2+\cdots+f_nx_n}{f_1+f_2+\cdots+f_n}.$$

II. **ASSUMED MEAN METHOD** – In the frequency table make a column for deviations $d_i = x_i - A$ about an arbitrary number A(assumed mean chosen from values of x_i),make a column for $f_i d_i$. **A**dd all $f_i d_i$ to get $\sum f_i d_i$, then

$$\bar{X} = A + \frac{1}{N}\sum_{I=1}^{n} f_i d_i \text{ , } where\ N = \sum_{i=1}^{n} f_i$$

III. **STEP DEVIATION METHOD** -In the frequency table make a column for $u_i = \frac{X_I - A}{h}$, where A = assumed mean and h = class size = upper limit – lower limit. Make a column for $f_i u_i$. **A**dd all $f_i u_i$ to get $\sum f_i u_i$, then calculate mean by using the formula

$$\bar{X} = A + \left[\frac{1}{N}\sum_{I=1}^{n} f_i u_i\right] \times h \text{ , } where\ N = \sum_{i=1}^{n} f_i$$

FOR A CONTINUOUS FREQUENCY DISTRIBUTION –

- First of all get the corresponding x_i as the mid values of each class interval , $x_i = \frac{upper\ class\ limit + lower\ class\ limit}{2}$,
- the calculation of x_i's to be done after making the class intervals continuous (by subtracting .5 from the lower class limit and adding .5 to the upper class limit).
- Then proceed to calculate the mean in the same way as done for discrete frequency distribution.

MEDIAN is the value of the variable which divides the distribution in two equal parts.

FOR INDIVIDUAL DATA – Arrange the data in ascending order. Count the total no. Of observations (n).

- If total no. of observation n is odd then median is the value of $\left(\frac{n+1}{2}\right)^{th}$ observation.
- If total no. of observation n is even then median is the mean of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observation.

FOR GROUPED DATA

- First make the column of cumulative frequency(cf) in the frequency distribution table

- Decide the median class as the class with cumulative frequency $\geq \frac{N}{2}$
- Calculate median by using the formula **median =** $\boldsymbol{l + \frac{\frac{N}{2} - cf}{f} \times h}$, where

  l = lower class limit of the median class,
  cf = the cumulative frequency of the class preceding the median class
  f = frequency of the median class
  h = the class size.
  N = total no. of observations

MODE is that value of the variable which has maximum frequency.

MODE OF A CONTINUOUS OR GROUPED FREQUENCY DISTRIBUTION

- First decide the modal class as the class corresponding to maximum frequency.
- Calculate mode by using the formula **mode =** $\boldsymbol{l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h,}$ where

  l = lower class limit of the medal class,

  f_0 = the frequency of the class preceding the modal class

  f_1 = frequency of the modal class

  f_2 = the frequency of the class succeeding the modal class

  h = the class size.

The three measures of central tendencies are connected by the relation :

Mode = 3 median – 2 mean

Cumulative frequency - of a class is obtained by adding the frequency of the class with all the frequencies of the preceding classes.

Cumulative frequency polygon and Cumulative frequency curve (ogive)

Less than method –

1. In the frequency distribution table make a column of 'less than'(x_i) by listing the upper limits of each class.
2. Make a column of cumulative frequencies (cf_i)by adding the previous frequencies with the class frequency.
3. Make a column of coordinates(x_i , cf_i), for each class take x coordinate as entry in step 1 and y coordinate as entry in step 2.
4. Mark the upper class limits (x_i) on the x - axis on a suitable scale.
5. Mark the cumulative frequencies (cf_i) on the y - axis on a suitable scale
6. Plot the points (x_i , cf_i).
7. Join the points by a smooth curve to get the **more than ogive** and join the points by line segments to get the cumulative frequency polygon

more than method –

1. In the frequency distribution table make a column of 'more than'(x_i) by listing the lower limits of each class.

2. Make a column of cumulative frequencies (cf_i)by subtracting the frequency of the class from the total frequency.
3. Make a column of coordinates(x_i , cf_i), for each class take x coordinate as entry in step 1 and y coordinate as entry in step 2.
4. Mark the lower class limits (x_i) on the x - axis on a suitable scale.
5. Mark the cumulative frequencies (cf_i) on the y - axis on a suitable scale
6. Plot the points (x_i , cf_i).
7. Join the points by a smooth curve to get the **more than ogive** and join the points by line segments to get the cumulative frequency polygon

   To obtain median from the graph –
   - Draw a line from the point (0, N/2) on y axis ,parallel to x axis .
   - Draw a perpendicular from the point where this line cuts the ogive.
   - Note down the x coordinate of the foot of this perpendicular. This is the required median.
   - If both the ogives are on the same graph then the x coordinate of the point of intersection of the more than and less than ogive gives the median of the distribution .

CH 15 – PROBABILITY

- **RANDOM EXPERIMENT** –A repeated controlled activity with well-defined outcomes which when repeated under identical conditions, do not produce the same outcome every time, but the outcome is one of the several possible outcomes
- **SAMPLE SPACE S** – the set of all possible outcomes of a random experiment is called the sample space.
- **ELEMENTARY EVENT E** – an outcome of a random experiment , i.e. n(E)=1
- **COMPOUND EVENT A** – an event obtained by combining two or more elementary events, i.e. n(A) > 1.
- **OCCURANCE OF AN EVENT A** – an event A associated with a random experiment occurs if the outcome is an elementary event associated with A.
- **PROBABILITY** of the happening of an event A is denoted by **P(A).**

$$\boldsymbol{P(A) = \frac{total\ number\ of\ outcomes\ favourable\ to\ the\ event\ A}{total\ number\ of\ possible\ outcomes} = \frac{n(A)}{n(S)}}$$

- Probability of non happening of an event A = P(Complementary of an event A) = **P(not A)** $=$ $\boldsymbol{P(\overline{A}\,)}$**= 1 – P(A)**
- **P(sure event) = 1**

- **P(impossible event) = 0**
- $\mathbf{0 \leq P(A) \leq 1}$
- **SUM OF THE PROBABILITIES OF ELEMENTARY EVENTS = 1**

  IN ORDER TO SOLVE A QUESTION

  I. Write down the experiment.
  II. Write down the total number of possible outcomes i.e. n(S).
  III. Write the event A.
  IV. Consider the total number of possible outcomes i.e. n(A).
  V. Then calculate the probability by using formula.

Best of luck!